THE MATHEMATICS TEACHER

The Official Journal of
The National Council of Teachers of Mathematics
Incorporated 1928

APRIL 1936

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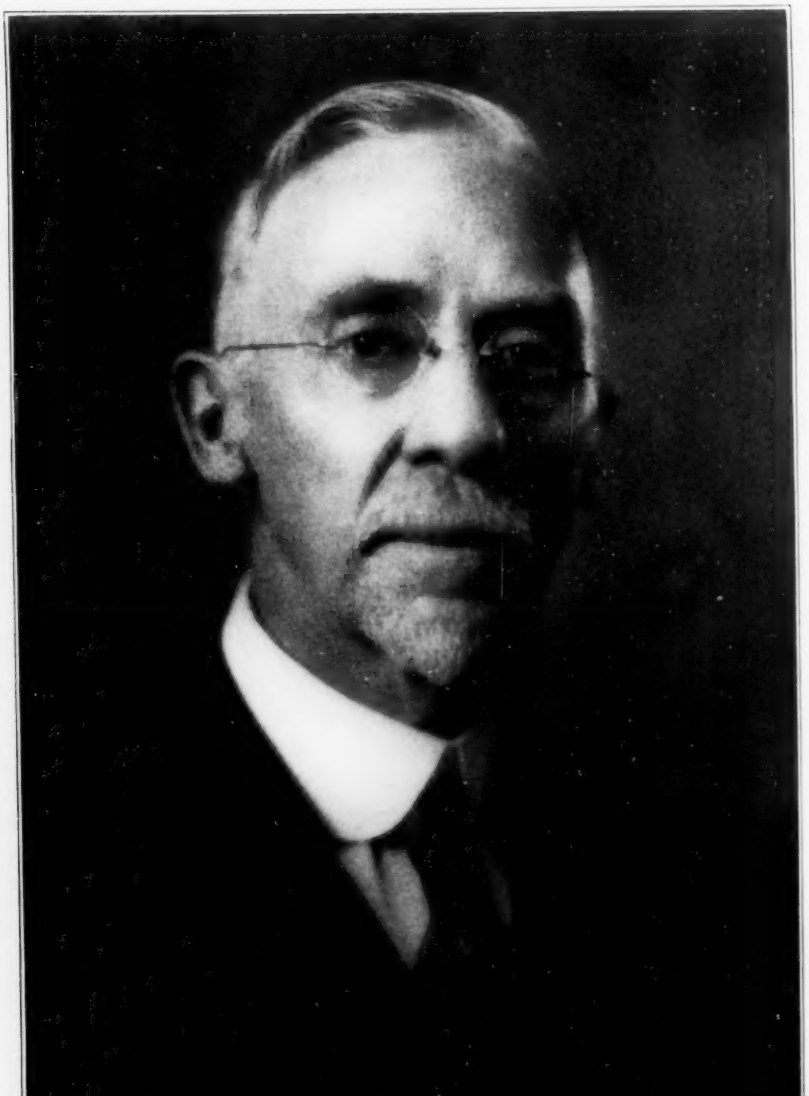
Volume XXIX **Number 4**

Published by the

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
MENASHA, WISCONSIN NEW YORK

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

THE MATHEMATICS TEACHER is published monthly except June, July, August and September. The subscription price is $2.00 per year. Single copies sell at 40 cents each.



THE MATHEMATICS TEACHER takes great pleasure in presenting this portrait of Professor Herbert E. Slaught who was recently elected honorary president of the National Council of Teachers of Mathematics.

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Volume XXIX Number 4

Edited by William David Reeve

Some Little Known Applications of Mathematics*

By HAROLD HOTELLING
Columbia University, New York, New York

I AM VERY much flattered by Professor Reeve's kind words, and by the invitation to speak to you tonight. I must disclaim one honor that Professor Reeve heaps upon me. I am not a professor of mathematics. I happen to be in the Economics Department at Columbia University. That is also relevant to what I have to say. I have taught mathematics at other places, and I am teaching mathematics now in the forms of mathematical statistics and mathematical economics.

The reason that is relevant to what I have to say tonight is that it serves as one minor illustration of the extreme degree of the generality of mathematics. This feature of mathematics has been a most distressing one to the numerous people who have an intensive passion for classification. They never know quite where to put mathematics or what to do with it. In the last two decades or so, in various colleges, to simplify administrative tasks, the different departments have been grouped into things sometimes called groups, sometimes divisions or schools. It seems there are too many departments for the comfort of presidents, so the presidents prefer to deal with deans, or committees, representing whole groups of departments. And there are always a number of very delicate questions as to how to group the departments. There is no department that is more perplexing in this respect than the Mathematics Department.

The problem has usually been solved by putting the Mathematics Department in the School of Physical Sciences. A great many people have the idea that mathematics is a physical science. They got that idea from the fact that a great many of the applications of higher mathematics have been in the domain of the physical sciences. The great discoveries that followed the appearance of calculus in the science of physics are known to every educated person. Sir Isaac Newton discovered the calculus, and through it deduced the principles of mechanics. The law of gravitation appeared only when it was possible to use the calculus to correlate it with the observed facts of astronomy. And in the centuries after Newton there was a remarkable efflorescence of applications of mathematics in the physical sciences, on the one hand, and, on the other hand, the development of new types of mathematics, many of which where suggested by the needs of

* A stenographic report of an address delivered by Professor Hotelling at the third dinner meeting of Section 19 (Mathematics) of the New York Society for the Experimental Study of Education, at the Men's Faculty Club of Columbia University on Saturday January 11, 1936.

physical sciences. It is on that basis that people very often think of mathematics as a branch of the physical sciences.

Some years ago I chanced to sit at the same luncheon table with a gentleman who is very well known to the American public. He has been engaged in a great educational experiment. He had devised a new scheme of college education which he thought would release the new generation from the trammels of hide-bound ideas and start off on an entirely fresh line of attack. He explained his plan, which he was then about to put into operation. The first year of college was to be built around that great historic episode, ancient Greece. Everything was to be correlated with the history of Greece—the science, the geography, the linguistics, all were to be tied to Greece. In the second year the curriculum was to be tied in the same way to another historical epoch and I presume by the time the students were seniors they might perhaps get down almost to the French Revolution. He told me something about the courses that had been designed, with the greatest of enthusiasm.

I listened, and then asked him, "Where does mathematics come into this? When and how are you going to teach mathematics to the students?"

"Well," he said, "we, as a matter of fact, are only in a preliminary stage of development, and the very special subjects we have had to defer for later consideration."

Now, the peculiar part of that was to regard mathematics as a very special subject. Mathematics, to my way of thinking, is the most general of all subjects. Everything else is more special than mathematics. There is nothing that has a richer profusion of applications, there is nothing that travels over the whole domain of human knowledge as does mathematics. There is no surer key to unlock all sorts of doors than mathematics. Mathematics won't get you all the way, but it will get you into places where you can't enter by any other method. It will supplement all other types of investigation, and help to get out profounder truths than would be possible without its aid. The remarkable thing is that whereas other methods of research are appropriate to special domains, to special kinds of investigations, mathematics—the same, identical mathematics—is applicable to this enormous profusion of different kinds of investigation and is adapted to bringing to light truths of the utmost diversity.

It may be appropriate to run over a few of the more important applications of mathematics. I leave out of account most of the well known ones, which are largely connected with the physical sciences, merely in order to give a little attention to some that are not well known.

It is true that in the physical sciences there are a great many applications and potential applications of mathematics that are very little known even to the people working in those sciences. Twenty-five years ago it was not at all uncommon for a professor of physics to say to one of his promising students, "You say you want to study mathematics. Well, mathematics is all right for those physicists who are not much good, but you, being a bright boy, really have some promise. You are likely to do something in physics, and therefore the thing for you to do is to concentrate all your time and energy on fixing up your apparatus so it will work, and leave out these mathematical refinements. If you can't get a thing by experiment it is not worth the time. Otherwise you won't have confidence in it anyway."

That generation of physicists no longer exists in that form. Some of them have passed out of the picture and the rest have changed their tune. Physics is now such a predominantly mathematical science as to put severe strains upon the mathematics that exists. Physics today is waiting in many departments for the development of additional mathematical theory that must be developed before the thing can get on. And not only physics, but many other subjects as well. In physics there are new applications of old branches of mathe-

matics, there are new applications of new branches of mathematics, and there are great vacancies where mathematics that is yet to be born will indubitably be applied.

To mention a few of the older and more familiar of these, there is the three-body problem. The two-body problem, of course, was solved mathematically by Newton after it was solved empirically by Kepler. Everyone knows the solution with two bodies moving about a common center of gravity. When you put in a third body, even in the most special cases—even if you simplify it down to supposing that you have an enormously large sun, and then a Jupiter much smaller, and then a satellite negligibly smaller than Jupiter, and then suppose Jupiter goes around in a circle, and the satellite moves in the same plane as the planet—supposing all those simplifications to be applied, the problem is still unsolved. A few still more special cases have been solved, and certain qualitative properties have been worked out; such as certain criteria, I believe still incomplete, as to when the satellite gets rejected to infinity, and when it hits the sun or Jupiter. But the general problem is still far from being solved. And when you go into the n-body problem you get into inextricable calculations. Those required for the problems of astronomy are carried out with great labor to a limited degree of approximation. They have to construct almanacs and ephemerides, so they calculate by means of series and hope they converge; but complete and general solutions, and properties of solutions which are of the utmost importance, remain to be found. The problem is connected, for example, with the stability of the solar system, its origin and destiny; it remains to be worked out.

Then, besides that rather classical type of problem, we have a lot of new ones. The theory of relativity has introduced complications of a remarkable sort. The absolute differential calculus, starting with the work of Gauss, and that of Riemann in 1854, led to work by Christoffel, in 1869, on the transformations of a quadratic differential form. This purely analytical approach was pushed further by Ricci and Levi-Cività in a paper published in 1901. Then Einstein came along and took it all over and used every bit of it in his theory of relativity, which is a branch of physics. Then other people were led to further developments of the same general type.

Now, quantum mechanics and wave mechanics have brought still more complications, and of quite a different sort. We now have a theory of matrices, square arrays of numbers, playing a part in wave mechanics, and also playing a part in psychology—the same general type of matrix theory. There are rules for the addition and multiplication of matrices. We have polynomials in matrices and infinite series expansions of matrices. In wave mechanics there are matrices of an infinite number of rows and columns, and the elements are imaginary exponentials which the "wave mechanic" proceeds to expand. The exponentials correspond to oscillations. So we get matrices that involve imaginary numbers expanded in a series, and they represent the waves that go on in what was formerly supposed to be the ether. The ether, of course, has more or less vanished from modern thought. The waves go on. Bertrand Russell, in expounding the subject, once wrote, "If you ask a physicist what the waves are in since there is no ether, he will reply that the waves are in his equations. The equations are in his head, from which we must not infer that the waves are in his head."

Analysis situs, the branch of mathematics that is concerned with the properties of figures that are invariant under all kinds of continuous deformations, has come to play a part in the discussion of orbits, for example in demonstrating the existence of closed orbits, and in many other connections. There are fascinating things about analysis situs, much of which could be taught to school children.

It is easy to see, for example, that if you

have a spherical balloon you can distort it continuously into an elipsoid or into a cube, or into any one of an infinite number of other figures. All those figures are, in the analysis situs sense, equivalent to your spherical balloon. If, however, you have a doughnut-shaped figure, say, an inner tube, and blow it up, no matter how hard you blow you can never blow it into a sphere. This is one of the most elementary of those differences which, in their higher reaches, applied to three-dimensional manifolds, give us remarkable existence theorems about closed orbits, the stability of the solar system, and other interesting problems.

There are important applications of mathematics in the physical sciences outside of physics proper and astronomy. In chemistry, differential equations are coming to play a prominent part, especially in physical chemistry. Group theory has been applied in recent years to a discussion of possible organic compounds. By manipulation of the groups representing relations among carbon and other atoms, people have been able to get new light on the possibilities of organic chemistry. The theory of groups is concerned with sets of elements having a rule of combination, so that when you apply this rule of combination to two of them in a certain order you will get a third element of the group. This third one may be identical with one of the two from which you started or it may not. This rule of combination is called multiplication, just to be brief and explicit, but it isn't the same as ordinary multiplication.

Consider, for instance, a rotation of 90°. If you have a square and rotate it in its plane through 90° you can put it right back in its original position. And there are four elements in the group of rotations of the square. The cube has a group of more possible rotations into itself. The group may or may not be commutative. A group is commutative if A times B is the same as B times A. That may not be true. For example, if A consists of rotating this chair which I hold in my hands through 90° about a vertical axis (demonstrating), then B may consist of rotating about an axis running from front to rear, 90°, like that (demonstrating). Now, if I do A and then do B there is a question whether I get the same result as if I do B and then A. I will try it. That is when I do A first (demonstrating) and this is when I do B first (demonstrating). Such distinctions come into questions of chemistry.

In geology we have various applications of mathematics that come to the front from time to time. There used to be some ferocious controversies about the age of the earth at meetings of scientific organizations. Following the idea of creation in the year 4004 B.C. the age of the earth was extended backward faster and faster after the theory of evolution came in. The work of Fourier, published about 1828, the mathematical theory of conduction of heat, gave a clue to the calculation of the age of the earth, especially when people believed in the nebular hypothesis. They used the geothermal gradient, the rate at which the temperature of the earth increases as one goes down in mines, and working with it arrived at the limit of the age of the earth, which, as I recall it, was about fifty million years. That was what one group of scientists did. Another group, working from the age of fossils, concluded that the earth must be at least a billion years old, and the two just couldn't agree. Finally, the discovery of radioactivity and its generation of heat led to a modification of the calculations extending backward the age of the earth. But mathematics of a rather advanced order appeared throughout that controversy.

Another thing in geology is the study of crystals. Donnay's work on crystal refraction leads to elliptic integrals, a matter of considerable mathematical interest.

In a whole group of biological sciences, there has been very extensive use of mathematics in a great many different connections. For one thing, in genetics, maps of genes and chromosomes have been traced

out through experiments, not on the germ cells in which these chromosomes are seen, but on the organisms growing out of the cells, by breeding fruit flies and classifying the offspring, watching the occurrence of frequencies, and deducing from these frequencies of occurrence the position of certain minute genes on the chromosomes, genes so small you can't see them in the microscope. All that has called for the development of a very great deal of mathematical research.

This leads one inevitably to statistics. It is through the theory of statistics that a great deal of the mathematics that is brought to bear upon the various fields comes in. The theory of statistics is a very extensive branch of mathematics, based on probability and serving to help one interpret the evidence that accumulates from observations, and also to serve one as a guide in the accumulation of observations in order that the evidence may be obtained. That latter function has not always been understood adequately. It is now coming to the fore. A recent book by R. A. Fisher, called "The Design of Experiments," emphasizes very much that point of view. It is really surprising that so much more can be learned from a given number of repetitions of an experiment, or a given number of replications, by the use of proper designs than could be learned by going ahead in what seems the obvious fashion.

For example, you may want to try out four varieties of potatoes to see if there is a difference in yields. Now, the most obvious thing would be to go ahead and plant four patches or four fields with your potatoes, one field of each kind of potato, and see which kind wins. As a matter of fact, after performing such an experiment as that, you would have virtually no more information than you had to begin with, because whichever kind of potato gave you the best yield might not do so the following year or in a repetition of the experiment. There is a very important underlying gradation of fertility in soils that is always there and cannot be detected by looking at the surface. Unless you are in a position to eliminate or make allowance for that, you cannot trust the result of an experiment. You get essentially no information whatsoever out of a single isolated experiment of that kind. You may have some sort of basis for guesswork but no scientific information. It is clearly necessary to replicate, to have a variety of potatoes grown not on one plot but on at least two, and better on more than one, in order to get some idea of the variation from plot to plot of the yield of the same variety of potatoes.

Now, in laying out these plots there is quite an art, which Fisher expounds in his book. You might want to plant your four varieties of potatoes on sixteen plots, say, arranged four by four, in a square. How are you going to arrange them? Now, you might say, "Let one row of four plots be planted in variety A, the next row in variety B, the next C, and the next in D," so that each variety stretches east to west across the field. The result of such an experiment will give no definite information simply because there is quite likely to be a gradation of fertility running from north to south. There is no way to eliminate it if you follow that plan. Similarly, you can't have the plots running north and south because there is a good chance of there being a fertility gradient running east and west. You might try running them along diagonally, but then you have the chance of a diagonal gradient of fertility, and still your result is not valid.

However, the problem is not insoluble. You can adopt what is called a Latin square layout; that is, you can follow the principle of planting a variety always in such a way that it never occurs twice in the same row or in the same column.

This principle in itself is not enough to specify fully the experiment. Any layout satisfying that condition would be a Latin square. There are many Latin squares. The number of Latin squares four by four is 576. The thing to do to get a valid ex-

periment, one with a result free from bias, is to select one of these 576 possible Latin squares at random, by the actual operation of the mechanism of games of chance, such as throwing dice or shuffling cards, and pick out your design in that way and plant potatoes accordingly, and when you get through you will have a *valid* result.

You may not have a significant result. To get a significant result you must have enough of variance among the yields of your varieties to overbalance the chance variation resulting from heterogeneous soil and other causes of variation. That is, there may be so small a real variation, and so big a chance variation, that you cannot find the real variation. It gets lost. A result is significant if the differences discovered are great enough, relative to their errors, so that you can assert with a high degree of confidence that so great an apparent difference could not arise purely as a result of chance in the absence of a true relationship.

A difference in yields, or an effect of any sort in a statistical or experimental study, can be real without being significant; it can be significant without being real; it can be valid without being significant, it can be significant without being valid. And your estimate, your method of arriving at the thing, may be valid when the effect is not real. Those are three different concepts—*validity*, *reality*, and *significance;* and they all have to be considered in a statistical investigation or in the laying out of an efficient experiment.

One soon gets into combinatorial mathematics in the design of an agricultural experiment, and the design of an agricultural field experiment is, after all, very much the same sort of a problem as the design of any experiment. Any experiment involving errors, and every experiment does involve errors, faces that same type of problem. There is the need of validity which can, in general, only be obtained by some degree of randomization, by the use of some element of chance. There is the matter of significance, the determination of which requires tests in terms of probability.

The calculation of probability from data is performed by methods which may be efficient or may be inefficient. The great triumph of recent development in mathematical statistics has been in the perfection of a considerable number of exact and efficient methods of computing probability from data in places where formerly very inefficient methods were used. For example, the method of tetrachoric correlation has often been applied to fourfold tables, in which people might be classified, for example, according to age and color of hair, to see if there is an effect of one thing on the other. You have a frequency in each class. The method of tetrachoric correlation is applied to calculate from such a table a coefficient of correlation which has then been compared with its probable error. This and many other such tests are now seen to be unnecessary. You can get a more accurate and a more efficient method of testing the same question from the same data by other more modern methods, which, incidentally, require less labor.

One great characteristic of modern mathematical statistics is that more accurate results are often obtained with less labor than by the use of methods which have been better known. Unfortunately, the newer results have not as yet found their way generally into textbooks. The teaching of statistics is still largely concerned with perpetuating the old mistakes copied from one textbook to another for the last twenty-five years. The same principles of design and methods of testing significance run through medical, agricultural and genetic experimentation that occur in problems of making borings to find out whether the geological conditions at a certain place will enable a dam to stand up under pressure. It is the same mathematics in all these cases, and in many others.

Turning to the social sciences we find a very considerable variety of applications

of statistical methods and also, as it happens, of other types of mathematics not involving statistics. Mathematics in economics is usually thought of as a matter of statistics. We have those prodigious acres of figures collected by governments and various organizations. Sometimes you can make something out of them and sometimes you can't. I suspect that there is no one who could ever get any information of value out of a great deal of the economic statistics that have been collected. What we need there is a thorough knowledge both of the relevant economic problems and of the necessary mathematics on the part of the people who decide what data shall be collected. It is very important to arrange these collections of data, these questionnaire and census schedules in such a way that it will be possible to get information out of the results when assembled. In preparing them, the same principles of the design of the experiment, the principle of paying attention to validity and to significance, must be observed. They have generally not been observed in the collection of economic statistics.

In economics there are also certain very important applications of mathematics that are not at all of a statistical character. At the very heart of economics, the fundamental thing, *economic theory*, has always proceeded in a quasi-mathematical fashion. There is, for example, a proposition, in which almost everybody believes, that if everyone is left to himself and will just pursue vigorously his own maximum profit, then everybody will be as well off as possible. That idea is very pervasive. Judges of the Supreme Court have always believed in it, and so have most other people. That doctrine is more or less vaguely ascribed to Adam Smith and God —chiefly God. If you ask a person to prove it, the best he can do is likely to be to take some very special case and speak of some particular sort of variation from these conditions of free, unregulated conduct of business that would make things worse. But to prove anything is a maximum or optimum you must also consider a wider class of variations, and to do that is a matter of calculus. Belief in this proposition of the efficacy of *laissez-faire* is very wide spread, and is much more popular than any reasoning that leads to it—anything that a mathematician would be willing to call reasoning.

I make a practice each year of presenting to my course in mathematical economics a mathematical demonstration of this proposition of *laissez-faire* as nearly as I can formulate it mathematically. The point in that is not to make people believe in the proposition—they believe in it any way—but to show what definitions and what assumptions have to be made in order to make a mathematical proof possible. By the time a person has understood the definitions and the assumptions involved in these proofs, he is quite willing to reject the result. You know people are always criticizing mathematicians on the ground that those who work with mathematics are inclined to place too much faith in a proposition merely because it is proved by mathematics. Nothing could be further from the truth. If you have followed through the demonstration of a proposition and understand it fully, you are likely to place very much less confidence in the applicability of that theorem to a particular situation than you would if you merely took the thing on faith. That is a very far-reaching principle.

In economic theory we can prove a great many things of vital importance to public policy by means of higher mathematics. One classical concern of economics has been the incidence of taxation. On the next page is a figure (Fig. 1) that used to appear, in spirit at least, in economics books. Now on the plea that the students are feeble-minded, it has dissolved into wordy pages. It shows relations between quantity and price of a commodity. You have a declining demand curve DA and a rising supply curve SA, also known as a marginal cost curve, and these intersect. The coordinates of the point of inter-

section (A) represent the actual price and quantity reached as a result of the operations of the market. Now, a tax, represented by the vertical stroke $BC = EG$, is placed upon the seller. He has to pay the government that much for every unit he sells. The effect is to shift the marginal cost curve upward. The new supply curve resulting intersects the demand curve at a

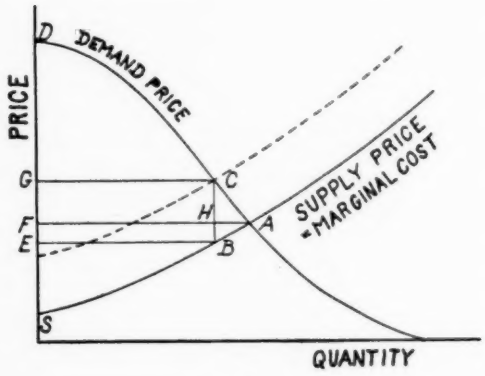


FIG. 1

new point above and to the left of the old. This is taken to demonstrate that the tax causes the buyer to pay a higher price and to buy a smaller quantity. The upward shift of the point is not by the full amount of the tax, but only by a fraction of it. The conclusion is drawn, and almost universally accepted, that the tax is divided between buyer and seller in a proportion depending upon the relative slopes of the demand and supply curves. That is the thing that almost everybody believes. The next more sophisticated thing in public policy, after the assumption that the tax falls where it is laid, is to use this figure to show that it is divided between buyer and seller.

There is one other proposition proved by means of the above figure. The benefit to consumers of a commodity, in the absence of the tax, is represented by the triangular area ADF, known as consumers' surplus—the total excess of what consumers would pay for a commodity, if they had to, above what they actually do pay. Likewise, there is a lower triangle AFS representing producers' surplus, otherwise known as profit. The total benefit from the commodity is represented by the sum of consumers' and producers' surpluses, or the large triangle ADS. Now the tax is laid on; the consumers' surplus shrinks to the small triangle CDG; the producers' profit shrinks to the lower triangle BES. The benefit now consists of the sum of the reduced areas of these triangles, and of the government revenue, which is represented by the rectangle $BCGE$. This is the tax per unit BC times the number of units sold under the new conditions, namely, EB. The total benefit is the sum of these three parts, which altogether fall short of the original benefit by this little triangle ABC, which represents a net dead loss as a result of the imposition of the tax. Since both the base BC and the altitude AH of this triangle tend to be proportional to the tax rate, the dead loss is approximately proportional to the square of the tax rate.

Now of these two propositions, the first, that a tax is divided between buyer and seller, has been very generally accepted. The other proposition, that there is a dead loss approximately proportional to the square of the tax rate has, however, been attacked by a great many economists on the ground that the consumers' surplus and producers' surplus are not satisfactory measures of utility, and because there are so many other commodities that are somehow related to any commodity you mention.

Now by the application of advanced calculus and of modern algebraic developments, it is possible, without too much trouble, to demonstrate two things: first, that the proposition that the tax is divided between buyer and seller, which everybody believes, is false; second, that the proposition that the tax causes a net loss approximately proportional to the square of the tax rate, which most people disbelieve, is essentially true. But you cannot prove it on the basis of a simple graph of this sort for the reason that the new proofs

consider a whole aggregate of commodities instead of single commodities. If you have as many as two commodities you cannot represent the supply or demand situation on a blackboard. You have two prices and two quantities, and if you are to have a picture, you have to go into four-dimensional space. Instead of demand and supply curves you have a demand surface and a supply surface—two surfaces, and the two meet, not in a curve, but in a point—one single point. Two planes, you know, in four-dimensional space, generally meet in only one point. To argue about that kind of thing visually is rather difficult for most people. It certainly is for me. To study four-dimensional or n-dimensional geometry requires us ordinarily to use analytic geometry, to use variable coordinates and equations. Now if you introduce variables and equations in order to study geometry, and geometry in order to study economics, you can save effort if you introduce the variables and equations to pertain directly to the economic quantities, omitting all the hard points of the geometry. This is a more modern way of handling these problems involving a plurality of commodities.

Another type of economic problem pertains to monopoly profit. You have problems of maxima and minima. When you try to get maximum profit or minimum cost, or maximum utility, subject to restraints, you have problems with Lagrange multipliers in them. If you have just a fixed income and you want a lot of things you can't afford to buy and have to choose among them you are in effect solving, either experimentally or in some other way, a problem in getting a maximum, subject to a limiting condition. If you happen to live in a country where rationing by government is in effect, you have not one restraint, your income restraint, but also others. These lead to problems in advanced calculus. We have to consider not only the first order conditions with the Lagrange multipliers, but the second order conditions. It is from these conditions on second derivatives that we obtain certain conditions on first derivatives of demand and supply functions, which generalize for whole groups of commodities the condition which the declining demand curve and the rising supply curve represent for a single commodity, and amend that condition, restating it in quite different form.

If we pass from the problems of static economics to dynamic problems, we have need for much more advanced mathematics. We leap at once from the differential calculus to differential equations; not only differential equations, but integral equations and functional equations which have been applied to the dynamic study of economics.

To take one type of economic situation, varying with time, on which I once published a paper, consider exhaustible resources, for example, oil, coal, copper. There is just a limited amount in the earth. When that is gone there won't be any more.

Suppose you own a mine. You want to decide how fast to work it. If you extract your material and put it on the market too rapidly, you will depress the market price perhaps to zero, and that, from your standpoint, is a very unwise thing to do. So you must not exploit the thing too rapidly. You also run up your cost of production besides depressing the selling price. On the other hand, if you exploit it too slowly you will be postponing profits into the indefinite future. In between there is some most profitable rate of exploitation. But as time advances that right way of exploitation will change. As exhaustion approaches it will cause the price to rise and the rate of production to diminish.

The determination of a *curve* of production then is called for. A curve of production such as AB (Fig. 2) representing the rate of production per unit of time will be chosen by the owner of the mine, subject to the limitation that the total area under the curve shall not exceed the amount of material there is in the mine, in such a

way as to maximize his net profit, allowing for interest and taxes.

Now, it is interesting to compare the curve of exploitation that will be adopted by a private monopoly controlling a resource with the curve of exploitation that would be adopted by a benevolent and all-wise state controlling the same resource under like conditions, with the same interest rate. The comparison of the two may surprise you. We have heard so much about conservation, about the wastefully rapid exploitation of natural resources, that we are inclined to leap to the conclusion that the wise and benevolent state would exploit more slowly. As a matter of fact, almost the reverse is true.

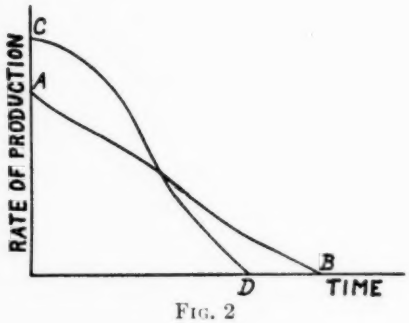


FIG. 2

If the monopoly's curve of exploitation comes down gradually, like AB, the state's curve of exploitation will come down less gradually—in the fashion of CD. The area under the two is the same. The thing is that the monopolist, to get his maximum profit, produces less, he limits production, whereas the state has no motive in limiting production just in order to keep the price up, and therefore produces more rapidly in the present, and consequently has less left in the remote future.

Under free competition, presumably, the same result will be obtained as under the regime of the benevolent and all-wise state. But free competition in large industries is impossible. Free competition in the railroad industry, for example, would mean that between here and Chicago you would have an infinite number of railroads competing vigorously with each other, the effect of which would be to bring down rates to such a point they couldn't possibly pay interest on their bonds.

If you look into the realms of human thought from which you might expect mathematics to be very far removed, you find still that it somehow or other finds its way in every now and then.

In history, which one considers as nonmathematical a subject as one could find, you discover various questions about chronology, about various temples, where the fine hand of mathematics has made its influence felt. According to accepted chronology there were seven kings of Rome in the early days, Tarquin kings, who reigned for a total of 350 years. Karl Pearson examined the question whether 50 years per king wasn't a good deal, for seven of them in a row. He went at the problem in two different ways. One way was with the help of an actuary and modern actuarial table; he came out with the result that it was perfectly incredible, even without taking account of the fact that few Roman kings died in bed and that their lives were cut short much more drastically than the lives of modern insured males. From another standpoint, Pearson examined the lengths of the reigns whose dates are accurately known, including such long reigns as that of Louis XIV, who reigned 72 years, and Queen Victoria, but also a great many short reigns, and on the basis of this frequency distribution Pearson again arrived at the conclusion that it was incredible that seven kings in a row should reign on an average of 50 years each. The revision of chronology has thus been made necessary by the theory of probability.

There are certain temples in Greece and Egypt which are supposed to have been oriented toward a certain star representing a deity. The star in rising would cast its beams down the length of the temple. Now, however, the star doesn't do that. The precession of the equinoxes has carried the star along so far that when the

star rises it throws its beams in another direction. By careful calculations, dates have been assigned to these temples, and I saw a list of the dates once which gave for each temple not only the year and day, but the hour and minute of construction.

In political science there is a very extensive field for the application of mathematics of various kinds. The study of elections raises many interesting questions as to the way to conduct an election so as to make the will of the voters felt as accurately as possible. One thing which is perfectly clear from these studies is that the ordinary system we have in this country is almost the worst possible system. That is, the system in which a mere plurality elects, with no requirement of a majority, is very likely to thwart the will of the majority. No third party gets a start for people say they will be throwing away their votes. "One of the two strongest ones will get in, and we will vote for the less undesirable of these two candidates."

In France they do things differently. There a majority is required to elect, and if there is no majority they have another election the next Sunday in which, I think, only the two leading candidates for each office appear. The result is that in France numerous political parties flourish. There is an opportunity for all sides to be heard, instead of a permanent and stable maintenance of a system in which there are always the same two parties representing the same identical point of view.

The constitution of the United States specifies that representatives and direct taxes shall be apportioned among the states in proportion to population as nearly as may be, and to carry that out provides for a census every ten years. Well, they have never tried to apportion any direct taxes, but they have tried to apportion representatives in Congress according to the population, and have had a most woeful time of it. As a matter of fact, it is impossible to apportion representatives in Congress among the states in proportion to the population because you would have to have fractional representatives. There is a question then, what criterion are you going to adopt?

But if you go about this problem of the apportionment of representatives, you may talk about the number of representatives per voter and try to make the sum of discrepancies of that character among the different states, or the sum of squares of discrepancies, a minimum, or talk about the number of voters per representative and try to minimize the sum or the sum of the squares of those discrepancies. There are also many other ways of carrying it out. The method used for years is the method known as the method of major fractions, which has nothing to recommend it except the fact that it has been used so long.

That method sometimes leads to peculiar results. For instance, it sometimes leads to the "Alabama paradox." The Alabama paradox is so named because it was first observed at a time when the apportionment of representatives was under discussion. The size of the House had not definitely been determined. They are always increasing the size of the House of Representatives to prevent disappointment to particular states, and especially to the representatives from those states who would lose their jobs if the number of representatives were decreased. The attempt to avoid that has led to a continual increase in the size of the House, which is most unfortunate. The Alabama paradox occurs when an increase of one in the size of the House will result in some state having its representation reduced. It was Alabama that got hit the first time. Any method that involves the Alabama paradox cannot be said to be a good method of apportionment of representatives. Professor E. V. Huntington has devised what he calls "the method of equal proportions," presented it to the American Mathematical Society, and urged it upon the appropriate committee of Congress. It seems to have a very good logical position. It has not, however, been adopted by Congress.

In journalism there are great opportunities for applications of mathematics and statistical methods. I happen to be a bachelor of arts in journalism of the University of Washington, and while a student there I undertook a piece of research, attempting to discover the influence of newspapers on elections. Over a ten-year period I took the statistics of municipal elections every year, which were of a nonpartisan character, with no party designations on the ballot. The newspapers were in the habit of printing advisory ballots and the people would read them and vote, and the question whether they voted according to the advisory ballot remained to be determined. So I set about discovering this from the statistics. Unfortunately, at the time my mathematics had not gone very far and I devised my own methods, which were very poor ones, for handling the data.

The morning paper said "Vote *Yes* on this bond issue," one of the afternoon papers said "Vote *No*," and the other afternoon paper said "Vote *Yes*." I set up an equation, $x-y+z=$ the majority in favor of the bond issue x, y, and z being the numbers of votes affected by these three papers. Take another bond issue and you get another equation. Now, if there were just three bond issues it would be very nice, for there would be just as many equations as unknowns, except that they might be dependent.

But in various ways I accumulated a large number of equations for each election, considering candidates, bond issues, and so forth, and the question was how to solve them. I naïvely divided into groups of three and solved every set of three and averaged the results. The labor was enormous, and the results were not all that might be desired. They did indicate that the morning paper went along at a very steady rate, as far as influence was concerned, in the ten years. The Scripps afternoon paper was high but irregular. The third paper (and it had the biggest circulation) had a curve of influence that started high in 1909, and then went down, until in 1915 it got down to minus 10,000 votes—after which it stopped printing advisory ballots. The study could have been made more accurately and with less labor by the method of least squares.

In the so-called science of law there have been many attempts to be really scientific, and these have usually led to the use of such mathematics as the people making the attempt were able to command. Condorcet, a very brilliant Frenchman who lived in the eighteenth century, tried to do all sorts of things that are impossible. One thing he tried to do was to attach probabilities to evidence and measure the strength of judgment. If A said that B said that C said that D was a liar, and if the probability of each of these fellows telling the truth is known, what is the probability that D told the truth? You will still find that kind of question, and Condorcet worked with it.

There have been recent attempts by legal writers on evidence to interpret their results in terms of probability. They can find a great deal that is relevant to their work in the modern developments of mathematical statistics.

If you pass from history, economics, political science, and law, over to literature, you may think you are safe from the mathematical demon. But even here mathematics creeps in. There is a famous controversy over the authorship of "La Farce de Maître Pathelin," a medieval French work. The problem has now, I understand, been pretty thoroughly resolved as a result of a lot of research work in the University of California, involving statistics, counts of certain combinations of words in the writings of the man who is now believed to be the author, compared with counts in the poem and counts in the writings of other competing authors.

Biblical exegesis has depended a great deal on internal evidence. The question of whether Paul wrote Hebrews is one which has been argued over a great deal, and I believe the current opinion is that he did

not. But in all such discussions, internal evidence, certain characterizations of literary style which may be put in mathematical form, become vital.

Then in psychology—I have left that out of account because I take it that everybody here knows something about it—in psychology of course, we are now having a great burst of mathematics. In a book that appeared within the last few months, Professor Thurstone's "Vectors of the Mind," a large part is essentially a very elementary text on matrix theory, the same matrix theory that occurs in higher algebra and in wave mechanics. And in the general problem of resolving mental tests into components we have a great deal of mathematics coming into the picture these days. Tetrad differences and many other phases of current psychological work are intimately related to very advanced mathematics.

All this suggests that there is really quite a bit of mathematics to learn. The educated man or woman of the coming generation cannot neglect to study mathematics. And the mathematics they study must not consist entirely of a lot of drill on multiplication and division. They can't put in all their time learning how to solve arithmetic problems that turn not on a knowledge of mathematics but on conventions, such as conventions about the width of wall paper, and methods used for carpeting rooms. We have got to cut out a lot of those conventions (most of them will be out of date anyhow by the time the students have grown up to use them), and put in their places some real knowledge of mathematics, higher mathematics, much more mathematics than has heretofore been thought necessary.

It is going to be necessary to introduce the calculus into the high school if men and women are to be equipped on the higher levels to meet many of the problems in the complex civilization in which we now find ourselves.

The pettifogging insistence on small things, on drill, chewing away on elementary practice in place of rapid progress into higher mathematics is really the great obstacle to development in a great many people of a higher mathematical culture.

The fate of a civilization that pays such close attention to this "chewing away," I suppose cannot be represented any better than in the case of one of the heroes of that delightful book of Hilaire Belloc, "Cautionary Tales for Children." These cautionary tales are of bad children who come to all sorts of dreadful ends. There is one Henry King who chewed bits of string and perished miserably. The little bits of string he chewed remind me very much of little bits of details, of applications of arithmetic that play so prominent a part in a good deal of elementary instruction.

The chief defect of Henry King
Was chewing little bits of string.
At last he swallowed some which tied
Itself in ugly knots inside.
Physicians of the utmost fame
Were called at once; but when they came
They answered, as they took their fees,
"There is no cure for this disease.
Henry will very soon be dead."
His parents stood about his bed
Lamenting his untimely death,
When Henry, with his latest breath,
Cried, "Oh, my friends, be warned by me,
That breakfast, dinner, luncheon, tea
Are all the human frame requires."
With that the wretched child expires.

The Permanence of Mathematics

In most sciences one generation tears down what another has built and what one has established another destroys. In Mathematics alone each generation builds a new story on the old structure.—Hankel.

Testing the Ability to Study

By Joseph B. Orleans
George Washington High School, New York City

Examinations have always been considered an essential element in the education of the young. From the earliest times, school routine has consisted of teaching by the teacher, learning by the pupil, recitation by the pupil and examination by the teacher. Under the monitorial system of instruction in the early part of the nineteenth century one of the duties of the monitor was to examine the pupil periodically and to promote him if he made progress. The lecture-quiz system in the colleges today is essentially an examination system. The recitation method still in vogue in many a classroom in our secondary schools is a form of the examination system. The pupil studies the lesson to which he has been exposed and is then subjected to a brief oral examination to show the teacher to what extent he has mastered the facts that have been taught. Under any system, examinations are held at stated intervals, usually at the end of ten weeks, at the end of fifteen weeks, at the end of a semester, in order that the teacher may measure the achievement of the pupils and record a mark to comply with the regulations of the schools.

During the past two decades, dissatisfaction with the traditional examination has led to the development of the movement of measurement in education together with the various new type tests. These tests for many reasons are an improvement over the old type essay examination, although they have weaknesses of their own. Fundamentally, however, all types of tests are supposed to have as their purposes (a) to inform pupils of their achievement, (b) to act as incentives to study, (c) to determine promotion, (d) to diagnose the work of the pupils, and (e) to determine the quality of instruction. A study of examination papers given in schools from term to term shows, however, that the questions aim at how much the pupils remember, to a lesser degree at how much they understand, and least of all at how much they can apply what they remember or understand.

One phase of instruction which examinations do not touch is the technique of learning. Teachers commonly bemoan the fact that their pupils do not know how to study; and, still, to what extent do teachers include this technique of learning in their teaching? This phase of instruction is as important as, if not more important than the mastery of the content of the subjects in the curriculum in the training of the vast majority of the children.

The writer, as a supervisor, is interested in the success of teachers in teaching their pupils to study mathematical material by themselves as well as their success in getting the pupils to repeat what they have seen demonstrated in class. Can the former be measured through an examination? During the past year, the writer has made an attempt in this direction in several grades of mathematics.

1. The following was included as a question in the examination in first term algebra in April 1934. The topic had not been taught in any of the classes.

Read the following explanation very carefully. Learn the method of doing the example. Then answer the question which follows the explanatory matter.

Given the problem: *The sum of two numbers is 8. Five times the first number added to three times the second number is 34. Find the numbers.*

Solution:

Let n represent the first number and s the second number. Then $n+s$ represents the sum of the two numbers. Therefore, $n+s=8$, since the problem says that the sum of the two numbers is 8.

Since $5n$ represents five times the first number and $3s$ represents three times the second number, then $5n+3s=34$ because

the problem says that five times the first number added to three times the second number is 34.

Consider together the two equations thus obtained:

Call	$n+s=8$	equation (1)
and call	$5n+3s=34$	equation (2)

From equation (1) you obtain $n=8-s$. Since $8-s$ equals n, it may be used in place of n. Therefore, in equation (2),

$5n+3s=34$ becomes	$5(8-s)+3s=34$	
or	$40-5s+3s=34$	
or	$6=2s$	
Therefore	$s=3$.	

Since you know now the value of s, namely 3, you may replace s by 3 inequation (1). Therefore,

$n+s=8$ becomes $n+3=8$ and $n=5$.

You have thus found that n equals 5 and s equals 3. Since the sum of 5 and 3 is 8, and since 5 times 5 added to 3 times 3 gives 34, your answers are correct.

Applying what is learned:

Using the method explained above, find the values of x and y in the equations

$$x-y=6$$
$$x+2y=24$$

The accompanying table gives the number of pupils who answered the question correctly, more than half correct sufficiently to show that they had grasped the method involved, and entirely wrong.

Teacher	Entirely correct	More correct than half	Entirely wrong	Total
A	75	25	69	169
B	35	15	79	129
C	27	8	57	92
D	19	4	18	41
E	4	6	27	37
Total	160	58	250	468

2. In the midterm examination in November 1934 one of the questions contained some explanatory matter about the addition of fractions including as illustrations the expressions

$$\frac{2}{3}+\frac{5}{6},\quad \frac{3}{5}+\frac{2}{3}-\frac{1}{2},\quad \frac{a}{2}+\frac{2a}{3}.$$

This was followed by the question: Applying what you have learned, express as a single fraction each of the following expressions:

$$\text{(a)}\ \frac{b}{5}+\frac{3b}{2},\quad \text{(b)}\ \frac{x}{4}-\frac{2x}{3}+\frac{5x}{2},\quad \text{(c)}\ \frac{1}{a}+\frac{1}{b}.$$

The following table gives the number of pupils by teachers who did each of the three examples either entirely correct or sufficiently correct to show that they had mastered the lesson:

Teacher	No. of pupils taught	No. of pupils who had mastered the lesson		
		a	b	c
A	140	70	63	23
B	21	18	16	2
D	111	51	39	1
Total	272	139	118	26

It should be noted that teacher *A* had among her 140 pupils one bright group of 27. Of these, 19 answered example *c* entirely correct. Teacher *D* had among her pupils a slow group of 30. Of these, none answered example *c* correctly.

3. The following was the midterm examination in second term algebra in November 1934. The explanatory matter in the various paragraphs had not been touched upon in class.

Read and study the following paragraphs very carefully. Then answer the questions that follow each paragraph.

I. *The world in which we live is a world of changes. Many of these changes are closely related. A study of such related changes is very important. For example, we all know that an increase or a decrease in the cost of food, clothing and other necessaries of life affects the welfare of millions of people. Every intelligent citizen follows the "ups and downs" in the prosperity of the community. Changing quantities are called variables. When two variables are so related that a change in the value of one of them causes a change in the value of the other, we say that one depends on*

the other. A quantity which does not change in a given situation is said to be constant. For example, the amount of money which an autoist pays out per week for gasoline depends upon the price of gasoline per gallon and also upon the number of gallons bought. The amount of money, therefore, increases or decreases as the number of gallons increases or decreases respectively and as the price per gallon increases or decreases respectively. The number of quarts in a gallon, however, is always four and does not vary. It is, therefore, constant.

EXERCISE 1

1. Write two illustrations of a quantity which you would consider a constant.
2. Write two statements, each illustrating how one quantity depends upon another.

II. *In the expression $y=x+3$, the value of y depends upon the value of x. As x changes in value, there is a corresponding change in the value of y. For example, if $x=2$, then $y=2+3$ or 5, if $x=5$, then $y=5+3$ or 8. As x increases, the value of y increases; as x decreases, the value of y decreases.*

EXERCISE 2

For each of these expressions, state whether y increases or decreases as the value of x increases:

(a) $y=2x$ (b) $y=x^2$

(c) $y=10-x$ (d) $y=\frac{1}{x}$

(e) $y=\frac{5}{4x+2}$ (f) $y=x-\frac{1}{x}$

III. *It is one of the chief purposes of science and of mathematics to study related changes, to find out the laws that govern them and to express the laws in a simple form. One of the tools for representing such related changes and for studying the dependence is the formula. For example, the formula $A=LW$ expresses the rule that the area (A) of a rectangular figure is obtained by multiplying its length (L) by its width (W). From this formula it is seen, for example, that if W remains unchanged, A will increase or decrease according to an increase or a decrease respectively in the value of L.*

Illustration: *If a train travels at the rate of 40 miles per hour, the distance (d) which it covers depends upon the number of hours (h) which it travels. The relation stated here is expressed by means of the formula $d=40h$.*

EXERCISE 3

1. If a carpenter earns $12 a day, write a formula expressing his wages (W) for n days.
2. An assembly hall has two sections, each containing r rows with s seats in each row. Write a formula for the total number of seats (n) in the hall.
3. If a newspaper costs 3 cents a copy, write a formula for the number of copies (n) that can be bought for x dollars.
4. To have a program printed, I had to pay $4 for the first 100 copies and 50 cents for each additional 100 copies. Write a formula for the total cost (c) of n papers, n being more than 100.

IV. *Another tool for expressing numerical relationships is the table. For example, in the formula $A=10W$,*
if $W=1, 2, 3, 4, 5, 6$, etc.
then $A=10, 20, 30, 40, 50, 60$, etc.
The table is read as follows: if $W=1$, then $A=10$; if $W=2$, then $A=20$, etc.

EXERCISE 4

1. Form a table of values for the equation $y=5x-3$ for the values of $x=-4, -1, 0, 3, 5$.
2. Form a table of values for the equation $2x+y=5$, letting x equal $-2, -1, 0, 1, 2$.

V. *Suppose that John earns two dollars a week while Henry earns six dollars a week. We can compare the earnings of the two boys in several ways. For example, we may say that (a) Henry earns four dollars more than John, or that (b) Henry earns three times as much as John. When we compare two numbers by stating how many times one number contains the other, we are using the ratio method. Thus, the ratio (relation) of 6 to 2 is 6/2 or 3. Similarly, the ratio of 20 to 15 is 20/15 or*

4/3 or 1-1/3. In general, the ratio of a to b is the quotient a/b. Ratios may be looked upon as fractions.

EXERCISE 5

1. For the following pairs of numbers, give the ratio of the first to the second:

   (a) 8, 4 (b) 7, 2
   (c) 4, 7 (d) ½, 4

2. If t and s are in the ratio 4 to 5, then

$$\frac{t}{s}=? \text{ and } \frac{s}{t}=?$$

3. In a crate of 126 oranges 14 are spoiled. What is the ratio of the number of good oranges to the total number in the crate?
4. A recipe for grape jelly calls for two pounds of grape juice, three and one-half pounds of sugar and a half pound of fruit pectin. What is the ratio of sugar to pectin?
5. Express the following ratios in the simplest form:

   (a) 75 to 100 (b) $2x^2$ to $6x^3$
   (c) a^2 to ab (d) 2^5 to 2^3

6. Two numbers in the ratio 3 to 4 may be represented by $3x$ and $4x$. Represent two numbers in the ratio:

   (a) 3 to 5 (b) 4 to 7 (c) 2 to 5.

7. Find two numbers whose sum is 100 and which are in the ratio 2 to 3.
8. A line 12 inches long is to be divided into two parts in the ratio 1 to 3. Find the lengths of the two parts.
9. Find three numbers whose sum is 1000 and whose ratio is 2 to 3 to 5.

VI. *It is evident that any fraction or ratio has many equivalent forms. Thus*

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{5}{10}=\frac{7}{14} \text{ etc.}$$

A statement expressing the equality of two ratios is called a proportion. Thus, $\frac{1}{2}=\frac{7}{14}$ *is a proportion. In the proportion* $\frac{a}{b}=\frac{c}{d}$, *multiplying both sides by bd, you get* $ad=bc$. *In the proportion, a and d are the outer terms and b and c are the inner terms. It follows from the illustration, therefore, that in a proportion the product of the outer terms equals the product of the inner terms. By means of this rule, you can find a missing term in a proportion.*

Thus, $\frac{5}{3}=\frac{x}{6}$ $\therefore 3x=30$ and $x=10$.

EXERCISE 6

Find the missing term in each of the following proportions:

1. $\frac{3}{4}=\frac{x}{20}$ 2. $\frac{7}{3}=\frac{x}{12}$
3. $\frac{5}{x}=\frac{9}{10}$ 4. $\frac{m}{n}=\frac{x}{c}$

VII. *If* $ax=bc$, *it follows through the reverse of the operation described in paragraph VI, that* $\frac{a}{b}=\frac{c}{x}$ *or* $\frac{a}{c}=\frac{b}{x}$ *or* $\frac{b}{a}=\frac{x}{c}$. *In other words an expression which states that the product of two numbers equals the product of two other numbers may be changed into a proportion by writing the factors of either product as the outer terms and the factors of the other product as the inner terms of the proportion.*

EXERCISE 7

Write each of the following in the form of a proportion:

1. $cy=mp$ 2. $x^2=bh$
3. $(a+2)(c-1)=ac$ 4. $a^2-b^2=5x$

The following table gives a summary of the scores by teachers in this examination. The maximum score was 66. The highest that any pupil attained was 63. This was the score on two papers. There was no score below 11.

Score	Teacher							Total
	B	C	D	E	F	G	M	
61–65	3				1	1		5
56–60	14		2	3			3	22
51–55	16		11	4	3	1	8	43
46–50	17	5	17	19	4	7	9	78
41–45	22	10	12	11	3	8	5	71
36–40	10	18	8	12	5	6	4	63
31–35	8	15	9	10	7	5	2	56
26–30	5	14		7	2	4	2	34
21–25		6	1	5	1	1		14
16–20		2	1	1	1	2		7
11–15		2	2		1			5
	95	72	63	72	28	35	33	398

4. The following was the midterm examination in first term geometry in April 1934. During the weeks preceding the examination the classes had covered the work involving congruence and parallelism that is normally found in Book I of the plane geometry texts.

NOTE: The explanatory matter in this paper deals with things which you have not been taught in class. Read it with great care. Try to understand it to the best of your ability and answer with care the questions that you will find throughout the paper.

A. *The curved line in fig. 1 is a circle. Point O is called the center of the circle. All points on a circle are equidistant from the center. Thus, in fig. 2, A is as far from O as B is from O. The distance from the center to any point on the circle is called a radius. The plural of radius is radii. The line connecting any two points on a circle and passing through the center is called a diameter.*

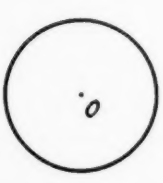


FIG. 1

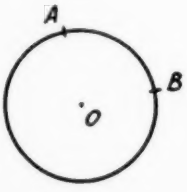


FIG. 2

1. How does *OV* (in fig. 3) compare in length with *OT*?
2. How does *ST* compare in length with *OV*?
3. How does the length of a diameter of a circle compare with that of a radius of the same circle?

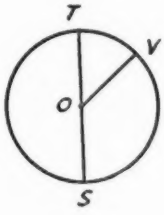


FIG. 3

4. How do radii of a circle compare in length?
5. If *TV* were drawn (in fig. 3) what relation would exist between angle *VTO* and angle *TVO*? Give the reason for your answer.

B. *Any part of a circle like the curved line XZY in fig. 4 is called an arc. The radii drawn from the ends of an arc form an angle whose vertex is naturally at the center of the circle. Such an angle is called a central angle. The central angle is said to intercept the arc that rests between the ends of the radii.*

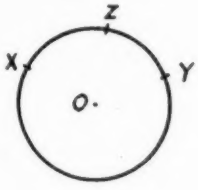


FIG. 4

1. Draw a circle in the space on the right.
2. Draw a central angle in the circle. Letter the diagram.
3. The central angle in the figure is .....
4. The arc which the central angle intercepts is .....

C. *It can be proved that in a circle, if two central angles are equal, they intercept equal arcs. Conversely, if two arcs in a circle are equal, the central angles which intercept them are equal. For example, in fig. 5 if angle m = angle n, then arc AB = arc CD. Likewise, if arc AB = arc CD, then angle m = angle n.*

1. If a circle is divided into four equal arcs and the radii are drawn to the

points of division of the circle, how large is each of the central angles formed? Draw a diagram to illustrate this statement and explain your answer.

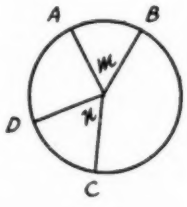


FIG. 5

2. If a circle is divided into six equal arcs and the radii are drawn to the points of division of the circle, how large is each of the central angles? Draw a diagram to illustrate this statement and explain your answer. Draw one line joining the ends of two radii in succession. What kind of a triangle is formed? Explain your answer.
3. In fig. 6 AB and FG are diameters. Name a pair of equal arcs. Tell why they are equal.

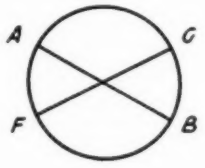


FIG. 6

4. In fig. 7, given arc RS = arc KL. Draw line RS and line KL. Prove: Triangle $ROS \cong$ triangle KOL.

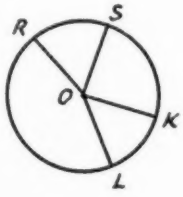


FIG. 7

D. *A straight line which connects the ends of an arc is called a chord. The chord is said to subtend the arc. Thus in fig. 8, FH is the chord which subtends arc FAH.*

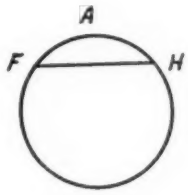


FIG. 8

Note that an arc *intercepts* a *central angle* and *subtends* a *chord.*

In fig. 9, arc EF = arc MN. You are going to compare chord EF with chord MN. Draw radii from E, F, M, and N. Two triangles appear in the figure. Prove the two triangles congruent.

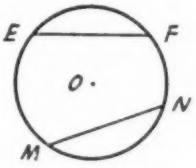


FIG. 9

What conclusion can you draw about chords EF and MN?

In fig. 10, chord RS = chord VW. You are going to compare arc RS with arc VW. Draw radii from R, S, V, and W. Two triangles appear in the figure. (Recall what you learned about equal arcs in a circle.) Prove the two triangles congruent.

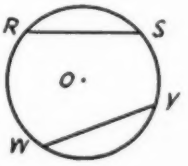


FIG. 10

What relationship can you find between the two arcs RS and VW?

In fig. 9 you proved that in a circle, *if chords are equal, they subtend equal arcs.*

In fig. 10 you proved that in a circle, *if arcs are equal, the chords which subtend them are equal.*

1. Triangle ABC in fig. 11 is equilateral. Show that the vertices of the triangle divide the circle into three equal arcs.

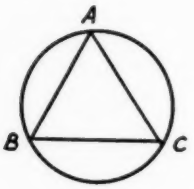


FIG. 11

2. In fig. 12, if arc EF = arc EG, what kind of a triangle is EFG? Why?

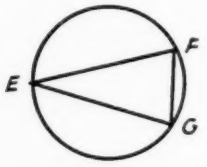


FIG. 12

3. From the midpoint of an arc perpendiculars are drawn to the radii which are drawn to the ends of the arc. Prove that these perpendiculars are equal. (Suggestion: Draw the radius to the midpoint of the arc.) See fig. 13.

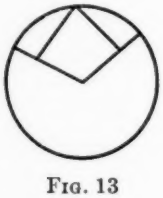


FIG. 13

E. *In the space on the right draw a circle with center O. Draw chord AB. Draw radius OC perpendicular to AB intersecting AB at E. Draw radii OA and OB. Prove triangle AOE congruent to triangle OEB.*

What conclusion can you draw about chord AB? Give reason for your answer.

What conclusion can you draw about arc ACB? Explain.

Extend radius CO so that diameter CD appears in the circle. Show that arc ADB is bisected at D.

You have just proven that if a line through the center of a circle is perpendicular to a chord, it bisects the chord and the arcs which the chord subtends.

1. *If two chords of a circle drawn from the same point of the circle make equal angles with the radius drawn from the point, then the chords are equal.*

Hint: From the center draw perpendiculars to the two chords.

F.

Given: *Chord AB = chord CD* (fig. 14.)

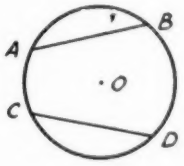


FIG. 14

Prove: *The distance from O to AB equals the distance from O to CD.*

Note: The *distance* from a point to a line is measured along the perpendicular from the point to the line.

(Suggestion: Draw radii OA and OC.)

Formulate a statement of the fact which you have proven in this exercise.

The following table gives the distribution of scores by teachers on this examination. The maximum score was 77. The highest that any pupil obtained was 75.

The writer attempts to draw no conclusions about the quality of instruction or the success of the teachers in teaching their pupils how to study. The reader may draw his own conclusions from the tables. The results do seem to indicate that a sufficiently large number of pupils show an ability in this direction which should induce the teachers "to put pupils on their own," to do less "teaching" so that the pupils may do more "learning" by themselves. F. W. Westaway, in his *Craftsmanship in the Teaching of Elementary Mathematics*, states the case very well when he says, "The one general conclusion that seems to emerge from the Dalton experiments is that pupils would do better if left to wrestle more for themselves, and that in the past we have tended to teach too much. Although the plan as a plan is,

in the estimation of not a few good judges, rather too revolutionary for general adoption, it must, on the other hand, be admitted that a clever teacher who loves teaching for its own sake may be something of a danger; he may do too much of the thinking, and leave the boys too little to do for themselves."

Score	Teacher									Total
	D	E	F	G	H	I	J	K	L	
70–75		1		2				1		4
66–70	4	4		1	1	2		2		14
61–65	7	3	1	1	6	4	2		2	26
56–60	3	8	2	12	3	7	3	2	12	52
51–55	6	15	3	6	10	14	4	4	6	68
46–50	4	14	6	10	12	10	10	6	12	84
41–45	1	9	5	6	11	4	5	4	15	60
36–40	4	8	4	5	7	11	8	1	4	52
31–35	4	11	3	12	5	3	10	2	5	55
26–30	1	17	3	3	9	3	7	2	7	52
21–25		7	2	8	2	9	5	1	1	35
16–20	1	3	3	5	4	2	10	2	5	35
11–15		2		4		1	5		1	13
6–10	1	4	1	1		1	3	2	2	15
1– 5		1	1	1			3	1		7
Total	36	107	34	77	70	71	75	30	72	572

Tangible Immortality

ONE THING that mathematics early imparts, unless hindered from so doing, is the idea that here, at last, is an immortality that is seemingly tangible—the immortality of a mathematical law. The student of algebra, for example, may well question the use of the traditional curriculum, but when he finds the value of $(a+b)^2$ he has come in contact with an eternal law. The laws of the Medes and Persians, unchangeable though they were thought to be, have all perished; the canons that bound Egyptian activities for thousands of year exist only in the ancient records, preserved in our museums of antiquity; the laws of Rome, which at one time dominated the legal world, have given place to modern codes; and the laws that we make to-day are certain to be changed to-morrow. But in the midst of all these changes it has ever been true, it is true to-day, it shall be true in all the future of this earth, and it is equally true throughout the universe whether in the algebra of Flatland or in that of the space in which we live, that $(a+b)^2 = a^2+2ab+b^2$.

We may change the symbols—they are temporary expedients to convey the idea; we may speak in different tongues—they are local expedients to convey thought; but it is inconceivable to us that the relation which the formula expresses should not be true always and everywhere—a tangible symbol of the immortality of law.

What I learned in chemistry, as a boy, seemed true at the time, but much of it to-day is known to be false. What I learned of molecular physics seems at the present time like children's stories, interesting but puerile. What we learn in history may be true in some degree, but is certain to be false in many particulars. So we may run the gamut of learning, and nowhere, save in mathematics alone, do we find that which stands as a tangible symbol of the immortality of law, true "yesterday, to-day, and forever."—From *Religio Mathematici* by David Eugene Smith. *The American Mathematical Monthly*, October, 1921.

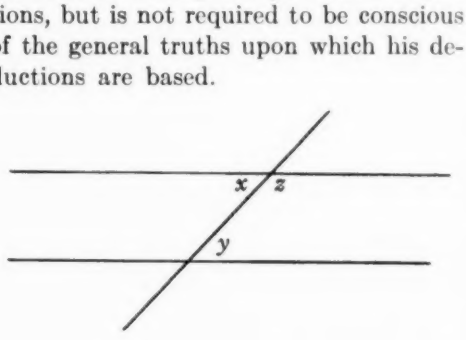
A Protest Against Informal Reasoning as an Approach to Demonstrative Geometry

By GERTRUDE HENDRIX
Eastern Illinois State Teachers College, Charleston, Illinois

THREE stages are commonly recognized in current recommendations for a secondary geometry course: (1) the intuitive stage, (2) the stage of informal reasoning,[1] and (3) the stage of formal reasoning, or demonstrative geometry. The second stage does not provide the transition to the third that is desirable. In fact, that informal reasoning has any value at all as an approach to the formal reasoning stage is questionable. An attempt to present reasons for doubts is set forth with care in the succeeding pages of this discussion. If the conclusion to which these reasons lead is valid, a decided change in the present methods of approach to demonstrative geometry should be effected.

All who believe in demonstrative geometry as an essential high school subject emphatically acclaim its power to "develop understanding and appreciation of a deductive proof and the ability to use this method of reasoning where it is applicable."[2] Educators have become insistent in pointing out the frequent absence of this outcome. Much of the deficiency has been laid to an inadequate approach. To remedy this weakness the National Committee on Mathematical Requirements in 1923, and many others since, have advised beginning geometry in an informal way at least as early as the seventh grade. It is now agreed that basic concepts should be developed intuitively by observation and experimentation. Similarly, the axioms, postulates, and the statements of many important theorems should be discovered inductively during the junior high school years. Then, as a further preparation for beginning demonstrative geometry, it is recommended that there be a stage of *informal* deductive reasoning.

In this discussion, informal reasoning is defined as *a deduction or series of deductions for which the reasons are not given*—that is, work in which the pupil makes deductions from given geometric conditions, but is not required to be conscious of the general truths upon which his deductions are based.



For example, a pupil is encouraged to say, "If $x=y$ and if $x+z=1$ st. $\angle$, then $y+z$ *has* to equal 1 st. $\angle$." If he is asked why, he probably answers, "Because $x=y$," and his answer is accepted. It is argued that early in the study of geometry it seems unnecessary to the pupil to state on what general truth his conclusion depends, and that it confuses and annoys him to insist upon mentioning the axiom, *A quantity may be substituted for its equal in any expression.*

There are two fallacious assumptions back of this whole idea that there should be a period of informal reasoning as a preparation for formal proof. They are:

1. That pupils beginning demonstrative

[1] This should not be confused with the "Stage B: Deductive Stage" referred to in *The Teaching of Geometry in Schools*, p. 15. London: Bell and Sons, Ltd., 1925. The stage of informal reasoning referred to in this article is a device meant for bridging the gap between stages A and B of the London report—that is, an approach to the process of proof.

[2] *The Reorganization of Mathematics in Secondary Education*, p. 48. Boston: Houghton Mifflin Company, 1927.

geometry usually lack sufficient experience in making deductions.

2. That need for deductive proof to remove doubt should be the primary motive for its introduction.

As to the first of these assumptions, observation reveals that deductive inferences are a common part of the average child's experience from a very early age. For instance, when a three-year-old boy, awakening from an afternoon nap, sees from his window that the family car is gone from the garage, he may infer that his father is away—a deduction of the informal reasoning type. In fact, someone has pointed out that most of the thinking of most people is deductive. The trouble is that so many of the average person's deductions are wrong. This is due not so much to his lack of experience in making them as to his failure to ask himself *upon what his deductions depend.* The child in the example above is unable to evaluate the reliability of his conclusion because he does not realize that it depends upon the faulty postulate, "When the car is gone, my father is away." Deductive thinking does not become reliable thinking until each step can be based consciously upon something else. In other words, the reliability of deductive inference can be checked only by *formal* reasoning.

Training in becoming conscious of this dependence of an inference upon something else is very difficult to direct except in the field of mathematics. In religion, politics, or the social sciences there is not sufficient agreement as to what constitutes fundamental general truths. Furthermore the assumptions are difficult to understand and complicated in statement. Even in the physical sciences argument must often be based upon postulates that are good today and refuted tomorrow. In mathematics, however, very early the pupil can be made conscious of undisputed principles upon which inferences may be made to depend. A typical example from fifth grade arithmetic may be found in reducing fractions to lower terms. In the intuitive stage a child discovers that $4/6 = \frac{2}{3}$ by dividing a whole into thirds, then into sixths, and counting the number of thirds in four-sixths. In the informal reasoning stage he may say $8/12 = \frac{2}{3}$ because he divided 8 and 12 by 4. However, this inference becomes reliable thinking only when he realizes that he has a right to say $8/12 = \frac{2}{3}$ because *dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction.* But as soon as this general principle is cited as the foundation of the statement the reasoning has become formal.

Now it becomes clear that no one can teach a child to do informal reasoning anyway. If a teacher attempts to help the pupils who have failed to deduce that $8/12 = \frac{2}{3}$ to make similar deductions in the future, she is driven to cite the principle. Until the pupil has felt the existence of the generalization and how it works, he must remain in the experimental stage, cutting a whole into thirds and twelfths and counting the number of thirds in 8 twelfths.

The assumption that there should be a stage of informal reasoning in geometry to provide experience in making deductions is fallacious, then, for two reasons:

1. The average pupil has already had extensive experience in making deductions; he needs instead, experience in checking the correctness of his deductions by consciously making them depend upon something else.

2. The below-average pupil who is poor in drawing deductive inferences from given situations cannot be helped to see *how* a deduction is made unless the assumption on which it depends is brought to light. The instant this is done, formal, not informal, reasoning is being taught.

The second fallacious assumption back of the recommendation that there be a period of informal reasoning in geometry was stated as follows: that need for deductive proof to remove doubt should be the primary motive for its introduction.

It is widely recommended that the teacher postpone the introduction of formal demonstration until the need for it arises out of a class argument or pupil skepticism. The trouble with this procedure is, that if a pupil does not already have the concept of proof, presenting one to him settles no doubt in his mind. The average class cannot absorb the significance of deductive proof quickly enough to meet an immediate need. Expecting them to do so seems as absurd as expecting them to do their first swimming upon being tossed into deep water. Just as one must learn to swim before the occasion to save himself from drowning arises, he must learn what deductive proof is before he can use it to meet an intellectual need.

In the light of these considerations it seems clear that preparation for demonstrative geometry demands not that the pupil have had much exercise in making deductions for which no reason was required, but that he have had extended practice in consciously making his deductions depend upon something else. In ohter words, instead of an informal reasoning stage, there should be substituted *widely distributed and widely extended practice in one-step formal proofs.*

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Economies Found in a Half Century's Teaching of Mathematics

By Jos. V. Collins
State Teachers College, Stevens Point, Wisconsin

Only a small fraction of those who teach hand down in written form the results of their experience. Other teachers affect future instruction only through the relatively small number of their students who afterwards become teachers. In this way there is a distinct loss to education and society itself. It occurred to me that my experience might have value, especially for young teachers, even though they may have read many books on mathematical instruction. But I know as well as anybody that many of the younger generation could give me hints for better teaching.

I taught twenty years using the mode of classroom instruction I had learned in Preparatory School and College. Then one day I came upon J. W. A. Young's book *The Teaching of Mathematics in the Higher Schools of Prussia*. I knew that Germany's schools had the reputation of being the best in the world. Ever since Napoleon's time the country had been striving to obtain the highest type of education possible, for one reason so that it might better protect itself if attacked in war. Young explained that the German teachers were making much use of what was called the "Chalk and Talk" method of classroom instruction. By this plan one member of the class is sent to the blackboard and required to solve a problem, give a demonstration, or make a construction, talking all the time he wrote, explaining what he was doing much as a teacher would. The members of the class watch what is being written and raise their hands in objection if a mistake or omission is made. The teacher sees that all mistakes overlooked by the class are promptly corrected. I tried the plan first with a Freshman class in college algebra, and found what might have been anticipated, that the work went forward very satisfactorily when a good student was reciting, but with exasperating slowness when a dull or illy prepared one was at the board. This was particularly true when the method was new to both teacher and class. But, notwithstanding the very slow progress at times, to my surprise I found that when the examination was held the students did much better than they had done before. I concluded afterwards that the very slowness which seemed so objectionable, gave the whole class an opportunity to keep up in all that was said and done. Later on I saved much time by not allowing students to recite, or even ask questions, who had not made proper preparation, and by giving fuller and prompter help to those who had tried and failed. This plan of instruction is particularly helpful to those who are poorly prepared, come in to the class late, or miss recitations.

Later I modified the plan by keeping about four students at the board all the time, others taking the place of those who finished. Young said that in the German school rooms only a small amount of board space was available, which may have accounted for one student at a time. Good students would have their work done and be ready to explain by the time they were reached. If not they had to use the chalk and talk way. Notice nearly the whole class hears all that is being said and sees what is being done and benefits by it, and constantly faces the idea that they themselves will be called on (rarely being excused) to go to the board and recite, which strongly tends to hold their interest and attention. Usually the questions would be copied on the board by a student all ready for the solution underneath, so that the student reciting had nothing to guide him save his knowledge of the

subject.[1] Thus he would be, so to speak, between the devil and the deep sea, the blackboard on one side and the teacher and class on the other. The reader may think this standard too high to work with his class. But after the class gets familiar with the plan, it works. In algebra or geometry if a forgotten process from arithmetic or algebra is required, a good student can be called on to give it and the class can copy the explanation in their notebooks. There is good psychology in this whole plan. The weaker students reason: If that student can know the lesson, I can too. Both eyes and ears of all are appealed to. The teacher is free to watch everybody and everything that is going on in the class, and also keep his record of recitations. If any members do not seem to be paying the best of attention, they can be called on to recite; or if they are bored by being forced to hear what they already know well, they can be sent to another part of the board to work difficult problems, which can be explained later. A good motto for a recitation is, Everybody on the job every minute and being benefited. Again, if a mistake is made, the teacher has a line on everybody in the class who does not hold up his hand. It should be understood that students should not be allowed to hold up their hands and offer to recite when the one called on hesitates or does not respond. Only if a mistake is made should the hands go up. Such objecting has a good effect on the whole class.

One of the great advantages of the mode of instruction just described is that the teacher acquires and keeps very definite information concerning the progress of every member of his class. In classes not of excessive size, every student should get a chance to go to the board and solve his problem at least every third day, usually much more often. To keep tab on students school authorities or teachers very often require written work to be handed in each day. When the student gets back such paper corrected, it is "cold." Generally he merely glances at it and the impression made is far weaker than that when the paper was written. Then the weak students are tempted to get help from other persons, taking their time, and getting no real benefit themselves. When this happens there is a threefold waste, and a moral injury. Moreover, writing is a slow procedure and the time of both student and teacher is consumed. When office and factory workers come to the end of the day their work usually stops till the next day. Not so with teachers who have to read written lessons at night. They should be studying to improve themselves or getting recreation. Teachers have plenty to do reading test and examination papers.

There is no thought here of limiting the mode of instruction to the Chalk and Talk plan. Thus the lecture, the heuristic, the laboratory, or any one of the plans frequently employed or any combination of them should be used when there is an advantage in so doing. It may be added if one of the unit or contract methods is being followed, the Chalk and Talk plan can be employed, since it clears up many difficulties the learner might meet in individual study.

Turning to algebra and the solution of equations, a great economy in the long run is secured by the rigorous teaching of the axioms, Here is a real difficulty and most authors and teachers are discouraged by it. When I first began teaching I discovered that not one but five hundred explanations of or references to the use of the axioms are needed for a good understanding. The drill should be like that on the multiplication table. The addition, subtraction, multiplication, division, power and root axioms are to be learned, first four at first. The procedure is taught in three stages:

(1) Arithmetic examples, as

[1] The situation is much the same in Germany today.—THE EDITOR.

$$\begin{array}{ll} 8=8 & 4=4 \\ 5=5; & \times 3=\times 3 \\ \hline 13=13 & 12=12 \end{array}$$

(2) Saying the axioms in the usual form (with word "equals") and at the same time *pointing* to the appropriate parts of the above arithmetic examples. Notice both hands have to be used, one for each side of an equation. This drill should be kept up till all four of the axioms are well learned, and their application to the arithmetic examples well understood.

(3) *a*. Transposition. A given algebra equation is written down and under it is put another containing all the terms to be transposed and a line drawn underneath it. The second equation is subtracted from the first, and after the resulting equation is written in parenthesis "Sub. Ax." The student now says the axiom and points to the appropriate parts of the different equations. All the time an arithmetic example is kept in the background.

b. The use of the other axioms is explained in the same way. Persistent drill is kept up till the use of all the axioms is well understood. Later the writing of the second equation underneath the first is discontinued.

Notice the writing of the axiom, as "Mult. Ax.," tells not only *what* has been done to the equation but also *why* one has a right to do it. This obviates the need for writing explanatory matter in solutions and improves the appearance of solutions.

I gave the axioms by number at first, but the late Professor E. B. Skinner of the University of Wisconsin proposed to me the use of the abbreviations as above, and I saw the point.

Now to the above explanation of the solution of equations there should be added the scales explanation and the equation balance explanation.[2]

[2] The writer described an inexpensive equation balance along with other like apparatus in *School Science and Mathematics* in the June 1907 number.

The economy in teaching equations in this way should be obvious. It does not take very long. It changes learning algebra from juggling with symbols into rational thinking. The learner is not upset when he comes upon a very simple or a very complex equation to be solved. Axioms are used seven times in the solution of most complete quadratics; manifestly here the student should know his axioms.

What economy or economies are there in the teaching of geometry? Probably the greatest lies in teaching logic. Every geometry teacher should be familiar with logic. Not that he should teach any of its intricacies. If a learner perceives in proving a proposition with a single figure that he is using induction and is passing from the one figure used to all such figures, it helps. If he is led to see that every time he quotes a proposition in the proof of another, he is employing the syllogism, (which he should have illustrated for him with examples outside of geometry), that also helps. The student sees the author always conscientiously proving converses, and wonders why. He should know that converses may or may not hold true. Thus if two triangles are congruent, two sides and the angle opposite one of them in one triangle equal the corresponding parts in the other, but the converse may or may not hold. Millions of men are everyday drawing false conclusions concerning important matters taking it for granted that converses hold true. Two well known authors of geometries proved propositions in Book II by quoting converses from Book I! Both corrected their texts. Even Jove nods sometimes. Geometry is the only application of logic we have below Senior college. A proper knowledge of it should have a blistering effect on common fallacies.

Probably every teacher of mathematics of long experience could go on almost *ad infinitum* telling of losses and economies from arithmetic to calculus and beyond. There is an enormous educational as well as economic loss because we do not use

the metric system. Our arithmetics would be only half as thick and that half only half as hard to learn and remember. Every student would be far better prepared to learn science. Every teacher of mathematics should be an active booster for the adoption of metric weights and measures. As matters now stand there is probably far more room for the introduction of economies in teaching college mathematics than in elementary and secondary mathematics. One important economy would result if authors would give enough of earlier branches used in solutions and proofs to enable the student to fully understand them. They should not be expected to know what has been dropped for a year or more.

Economies is now and has been for long a watchword in business and industry. Would that the idea could be carried vigorously into education.

A Message from Honorary President H. E. Slaught*

SIXTEEN years ago when the National Committee disbanded, its inspired leader, the late Professor J. W. Young, expressed the hope that representatives of the hundred or more clubs and mathematical organizations in the high schools throughout the country, whose loyal cooperation has been so helpful to the Committee, the hope, I say, that these same workers and many others would band themselves together to perpetuate the spirit of that Committee's remarkable report.

Under the inspiration of this request the National Council of Teachers of Mathematics was organized and the *The Mathematics Teacher* eventually became its official journal. The membership in the Council has gradually grown to over 4,000 and its worth and influence have been tested and proved in scores of situa-

* EDITOR'S NOTE

A number of Dr. Slaught's friends have wondered how he came to be so much interested in the secondary situation, when he had so many alluring attractions to occupy his attention in the collegiate and graduate fields while holding his professorship (1892–1932) at the University of Chicago. We have induced him to give us a few facts bearing on the answer to this question. We have outlined these below:

1. His first teaching (1883–1892) was in a secondary school where he completely reorganized the mathematical department.

2. From his early experiences at the University of Chicago which included supervision of entrance examinations and official visiting of secondary schools, he obtained ample evidence of the need of more effective training in high school mathematics.

3. For a long term of years, the University held an annual conference with cooperating high schools and Dr. Slaught had full responsibility for the conduct of the mathematics section, which brought him in close contact with the schools.

4. A year of graduate study in Germany (1902–1903) gave him fine opportunity, on the side, for observing the thorough methods in use in the schools there.

5. Upon his return in 1903, the Central Association of Science and Mathematics Teachers was just getting under way and he at once allied himself with that organization and became one of its most active supporters. (He is now one of three or four honorary life members.)

6. When the Mathematical Association of America was organized (1916) primarily for operation in the Collegiate field, Dr. Slaught was one of the first to insist on *Cooperation* with the secondary schools through the joint National Committee and this led to the organization of the National Council of Teachers of Mathematics, as explained in his message above.

7. It will thus be seen that Dr. Slaught has practised what he preached. He believes that the interests of mathematics can best be served not by putting up a wall of partition between the secondary school and the college, but by cooperation wherever possible among the representatives of the respective organizations.

8. Dr. Slaught's recent election as honorary president of the National Council of Teachers of Mathematics is an honor which he has well earned by his devotion to our cause.

tions, not least of which is the ground so admirably covered by the series of Year Books published by the Council (now eleven in number). The latest evidence of dynamic vitality shown by the Council is the organization of a joint Commission, together with the Mathematical Association of America, for the purpose of surveying the whole mathematical field, elementary, secondary, and collegiate and making such recommendations as seem wise in the present crises.

All these find accomplishments, and many more, lead us to wonder why the membership in the National Council is so small compared to the number of teachers of mathematics in secondary schools. I wonder how many of these would wish to depend upon a doctor who had no connection with scientific organizations in his field; and yet it appears that thousands are trusting their children to the care of teachers who have no such scientific connections.

What can we do to remedy the situation in our case? We can call attention to the National Council in season and out of season, wherever opportunity offers. In particular you can tear out the application blank from some copy of *The Mathematics Teacher* and hand it to a non-member in your group with a personal invitation from you to join.

If you have no mathematics club in your school or in your town you may be instrumental in forming such an organization. Thus you may become a real force in your community in upholding the interests of mathematics. In making such a start one may well combine social attractions with the more serious objects of the meetings.

At a meeting of the National Council some years ago, before the depression, I proposed as a slogan "Ten thousand members by 1935." This was somewhat too optimistic, as matters turned out. But why not change 1935 to 1940 and try it again? Let us deluge the business office for application blanks and use them on our friends and on those who should become our friends through personal contacts in the Council.

By such energetic measures we may not only increase the membership of the Council, but we may add to its financial resources which are necessary for carrying on the work of improving the standards of mathematical teaching and important research.

Eleventh Yearbook Ready Soon

THE eleventh yearbook of the National Council of Teachers of Mathematics on *The Place of Mathematics in Modern Education* will be ready for mailing soon. $1.75 postpaid.

Send all orders to The Bureau of Publications, Teachers College, Columbia University, 525 West 120th Street, New York, N. Y.

The 17th Annual Meeting of the National Council of Teachers of Mathematics

By Edwin W. Schreiber, *Secretary*

The Seventeenth Annual Meeting of the National Council of Teachers of Mathematics was held in St. Louis, Missouri, December 31, 1935 to January 1, 1936. This is the first annual meeting the National Council has held with the A.A.A.S. One hundred eighty-four registered for the meetings though the total attendance was well in excess of two hundred. A joint session with Section A of the A.A.A.S., the American Mathematical Society, and the Mathematical Association of America, was held on Tuesday morning, December 31, with approximately 250 in attendance. Professor Kenneth P. Williams of Indiana University presented a temporary report of the Joint Commission on the Place of Mathematics in the Secondary School. "The Main Purposes and Objectives in Teaching High School Mathematics" was discussed by William Betz of Rochester, New York, representing the National Council, and W. W. Hart, representing the Mathematical Association of America. On Tuesday afternoon the National Council presented a Symposium on the Teaching of Geometry. Professor W. H. Roever of Washington University, St. Louis, discussed in a very thorough manner the "Purpose, Nature, and use of Pictures in the Teaching of Solid Geometry." John T. Rule, the Taylor School, Clayton, Missouri, presented an interesting paper on "Stereoscopy as an Aid to the Teaching of Solid Geometry." The session closed with a stimulating discussion by Rolland R. Smith, Classical High School, Springfield, Mass., on "Developing the Meaning of Demonstration in Geometry." The Tuesday evening session was opened by an address of welcome by the Rev. Father Robert S. Johnston, President of St. Louis University. The response was made by Miss Edith Woolsey of Minneapolis, Minnesota. Professor Edwin W. Schreiber, State Teachers College, Macomb, Illinois, presented an illustrated lecture on "The History of the Development of the Metric System." Miss Ruth Lane, University High School, Iowa City, Iowa, presented an illuminating paper on "Mathematical Recreations, an Aid or a Relief?" On Wednesday morning, January 1, the Annual Business session of the National Council was held. At this session Professor H. E. Slaught of the University of Chicago was honored in being elected Honorary President of the National Council. Secretary Schreiber as Chairman of the Ballot Committee announced the results of the annual election: President—Miss Martha Hildebrandt, Proviso Township High School, Maywood, Illinois; second Vice President—Miss Mary Kelly, Wichita, Kansas; three new members of the Board of Directors—E. R. Breslich, Chicago, Illinois, Leonard D. Haertter, Clayton, Missouri, and Virgil S. Mallory, Montclair, New Jersey. The morning session closed with two interesting papers: "Functional Thinking and Teaching in Secondary School Mathematics" by Professor H. C. Christofferson, Miami University, Oxford, Ohio; and "The Crisis in Mathematics—at Home and Abroad" by Professor William D. Reeve, Teachers College, Columbia University. A stimulating session was the Luncheon Meeting for the Official Delegates present as well as all State Representatives present. The Annual Banquet was held Wednesday evening at the Coronado Hotel. Professor Raymond Clare Archibald of Brown University was the guest speaker. The title of his address was "Babylonian Mathematics, with Special Reference to Recent Discoveries." At the conclusion of the able

address, which was enjoyed by members of the A.M.A. and the M.A.A., as well as the National Council, President J. O. Hassler presented to the group the new President of the National Council—Miss Martha Hildebrandt, Proviso Township High School, Maywood, Illinois.

Meeting of the Board of Directors
Hotel Coronado, Parlor A
St. Louis, Missouri

11:55 A.M., Tuesday, December 31, 1935

The meeting was called to order by President J. O. Hassler of the University of Oklahoma. The following members of the Board of Directors were present: J. O. Hassler, Allan R. Congdon, Florence Brooks Miller, Edwin W. Schreiber, William D. Reeve, Martha Hildebrandt, W. S. Schlauch, William Betz, H. C. Christofferson, Edith Woolsey, M. L. Hartung, Mary A. Potter and Rolland R. Smith. Absent: Vera Sanford, Herbert E. Slaught and Harry C. Barber.

It was moved by Secretary Schreiber seconded by Editor Reeve that Herbert E. Slaught of the University of Chicago be elected Honorary President of the National Council of Teachers of Mathematics because of his loyal support and sincere interest in our organization.

Mr. Betz of Rochester presented to the Board of Directors Dr. A. M. Macmahon of the Museum of Science and Industry, Chicago, Illinois, who reported to our Board of Directors the matter of reproducing 5,000 copies of the *Tree of Knowledge* which was so effectively displayed at the Chicago Century of Progress Exposition 1933–34. Dr. Macmahon announced that if 5,000 copies could be printed in 4 colors they could be distributed at the modest sum of approximately 35¢ per copy. He also presented to the Board blue prints of the prospective layout of several rooms in the Museum of Science and Industry to be devoted to Mathematics.

The Board of Directors then temporarily adjourned to the Pine Room where luncheon was served. During the course of the luncheon President Hassler made his President's report to the Board of Directors. Treasurer Schreiber made a brief report on the state of finances of the National Council announcing that we were living within our budget. Vice President Congdon made a brief report on the Denver meeting which was supplemented by Edith Woolsey, Mary Potter and Martha Hildebrandt, these four having attended the first summer meeting of the National Council held in Denver, Colorado, June 29 to July 1, 1935. Editor Reeve made a brief report concerning *The Mathematics Teacher* and the forthcoming *Eleventh Yearbook*. Vice President Florence Brooks Miller presented her report to the Board on Affiliated Organizations and State Representatives. She displayed a very interesting map of the United States indicating thereon by means of dots the membership in the National Council for each state. The meeting adjourned.

Annual Business Meeting of the
National Council
Hotel Coronado, Crystal Room

9:00 to 10:30 A.M., Wednesday,
January 1, 1936

The meeting was called to order by President J. O. Hassler who asked Secretary Schreiber to read the minutes of The last annual meeting. It was moved and carried that the minutes be approved as printed in the May 1935 issue of *The Mathematics Teacher*. President Hassler then read his President's report which was approved as read. A vote of thanks for President Hassler's work as the chief officer of the National Council was unanimously approved. Treasurer Schreiber then made a brief report on the state of finances of the National Council announcing that we were living within our budget. Dr. C. M. Stokes of Temple University, Philadelphia, made a brief report for the sub-committee on Individual Differences. It was moved and unanimously carried that Herbert E. Slaught of the University

of Chicago be elected Honorary President of the National Council of Teachers of Mathematics and the secretary was instructed to wire Professor Slaught the good news. Editor Reeve of Teachers College Columbia University, presented his report concerning *The Mathematics Teacher* and the Yearbooks. President Hassler then read a telegram from the Tulsa, Oklahoma, mathematics teachers announcing that a new branch of the National Council is to be established in Tulsa. A message was also read from New Mexico asking for information concerning the establishment of a branch in that state. Secretary Schreiber as Chairman of the Ballot Committee announced the results of the annual election. For President: Martha Hildebrandt 133, W. S. Schlauch 108. For Second Vice President: J. T. Johnson 110, Mary Kelly 129. For Members of the Board of Directors: E. R. Breslich 175, H. W. Charlesworth 94, Leonard D. Haertter 120, Olive A. Kee 104, Virgil S. Mallory 121, Walter O. Shriner 69. Secretary Schreiber then declared the following elected: For President, 1936 to 1938, Miss Martha Hildebrandt, Proviso Township High School, Maywood, Illinois. For second Vice President, 1936 to 1938, Miss Mary Kelly, East High School, Wichita, Kansas. For members of the Board of Directors, 1936 to 1939, E. R. Breslich, University of Chicago, Chicago, Illinois; Leonard D. Haertter, John Burroughs School, Clayton, Missouri; Virgil S. Mallory, State Teachers College, Montclair, New Jersey.

Mr. Rolland R. Smith presented a communication from the Amherst Conference of Private School Teachers. President Hassler read a report from C. Louis Thiel of Detroit concerning the establishment of a National Committee on Arithmetic. He also read telegrams from Professors Wilson and Brownell. Professor E. H. Taylor moved (seconded by W. D. Reeve) that the Board of Directors consider the establishment of a National Committee on Arithmetic. Carried.

Vice President Congdon reported to the group briefly on the Denver meeting of the National Council and called upon Miss Mary A. Potter, Miss Edith Woolsey, Miss Martha Hildebrandt, and Mr. H. W. Charlesworth to give the group a complete picture of what transpired at the Denver meeting. The meeting then adjourned.

Meeting of the Board of Directors
Hotel Coronado, Room 845

3:30 to 5:30 P.M., Wednesday, January 1, 1936

PRESENT: J. O. Hassler, Allan R. Congdon, Florence Brooks Miller, Edwin W. Schreiber, W. D. Reeve, Martha Hildebrandt, W. S. Schlauch, William Betz, H. C. Christofferson, Edith Woolsey, M. L. Hartung, Mary A. Potter, Rolland R. Smith, Virgil S. Mallory, Leonard D. Haertter.

ABSENT: Vera Sanford, H. E. Slaught, Mary Kelly, Harry C. Barber, E. R. Breslich.

It was moved by Mr. Christofferson, seconded by Mr. Schlauch that the President appoint a committee to investigate the advisability and the feasibility of getting out a new register of members. Carried. It was moved by Secretary Schreiber and seconded by Mr. Schlauch that the Board of Directors of the National Council continue its policy of allowing no commercial exhibits at any meeting of the Council. Carried. A brief report on the Joint Commission was presented by Mr. Betz, Miss Potter, and Mr. Hartung. It was moved by Mr. Christofferson, seconded by Mr. Betz that the study of elementary mathematics be referred to the Joint Commission. Carried. It was moved by Secretary Schreiber, seconded by Miss Woolsey that as a matter of policy the finance committee which is composed of three members by our constitution be chosen as follows: One member from the officers, one member from the committee on the Official Journal and one member from additional members of the Board of

Directors. Carried. It was moved by Editor Reeve and seconded by Miss Potter that the President investigate the matter of affiliating with the N.E.A. Summer meetings without losing our identity. Carried. It was moved by Mr. Christofferson, seconded by Miss Woolsey that if the National Council holds three meetings in three different cities that the 18th annual meeting shall be in February at the time of the Department of Superintendence of the N.E.A. Carried. It was moved by Mr. Christofferson, seconded by Miss Woolsey that if the National Council holds only two meetings during 1936 that the 18th annual meeting be in December at the time of the meetings of the American Association for the Advancement of Science. It was moved by Editor Reeve, seconded by Secretary Schreiber that W. S. Schlauch be a member of the committee on Official Journal to fill the vacancy created by H. E. Slaught who was made Honorary President of the National Council. Carried. Considerable discussion was offered in connection with the Portland meeting during the summer of 1936 but no official action was taken. The meeting was adjourned.

Attendance

Alabama (1)
Athens
Wyant, Kathryn (G) Athens College
California (1)
Los Angeles
Hedrick, E. R. (R) University of California at Los Angeles
Colorado (4)
Boulder
Hutchinson, Chas. (R) University of Colorado.
Colorado Springs
Sesam, Charles (G) Colorado College
Denver
Charlesworth, H. W. (R) East High School
Loretto
Sister Rose Margaret Cook (W) Loretto Heights College
District of Columbia (1)
Washington
Grubbs, Ethel (B) High Schools—Divisions 10–13
Illinois (49)
Aurora
Steinbrecher, Francis (W) Marmion Military Academy
Belleville
Hexter, E. G. (W) Township High School
Bloomington
Wykle, Bertha (W) Bloomington High School
Bunker Hill
Whitfield, Clara (W) Bunker Hill High School
Carbondale
Goings, Victor (W) Community High School
Mayor, John (W) Southern Illinois Normal University
Wright, Alice (W) Southern Illinois Normal University
Carlinville
Kasteen, Alvarieta (G) High School
Centralia
Huck, Raymond (W) Centralia Township High School
Charleston
Hendrix, Gertrude (W) Eastern Illinois State Teachers College
Taylor, E. H. (B) Eastern Illinois State Teachers College
Chicago
Hawkins, G. E. (W) University High School
Johnson, John T. (B) Chicago Normal School
Krebill, A. P. (G) Austin, High School
Petty, D. Talmage (R) Francis W. Parker School
Shoesmith, Beulah (R) Hyde Park High School
Chicago Heights
Martin, Dorothy (W) Bloom Township High School
Clinton
Foote, Frances (W) Clinton High School
Marshall, Opal (W) High School
Nebel, Dora (W) Clinton High School
De Kalb
Hellmich, E. W. (R) Northern Illinois State Teachers College
Evanston
Moulton, E. J. (R) Northwestern University
Murphy, Clara (R) Evanston Township High School
Freeport
Martin, Mary C. (R) Freeport High School
Godfrey
Filkerson, Mary (W) Monticello College
Jacksonsville
Sister Mary Gerard (W) Routt High School
Lawrenceville
Roth, Selma (W) Lawrenceville High School
Lebanon
Barr, Fannie (W) Lebanon High School
Lincoln
Henneberry, Theresa (W) Lincoln High School
Litchfield
Kasten, Buenieta (G) Litchfield High School
Macomb
Ginnings, R. M. (W) Western Illinois State Teachers College
Schreiber, Edwin W. (B) Western Illinois State Teachers College
Maywood
Hildebrandt, Martha (B) Proviso Township High School

Metropolis
Wicicawer, Gladys (W) Metropolis High School
Monticello
Fleming, Mildred (G) Monticello High School
McKenzie, Betty (G) Monticello High School
Seybold, Anice (W) Monticello High School
Normal
Atkin, Edith (R) Illinois State Normal University
Flagg, Elinor (R) Illinois State Normal University
Mills, C. N. (R) Illinois State Normal University
Oak Park
Austin, C. M. (B) High School
Booher, Eleanor (R) High School
Johnson, Elsie Parker (B) High School
Ottawa
Le May, Mary (W) Ottawa Township High School
Springfield
Sister M. Mercedes (G) Sacred Heart Academy
Urbana
Taylor, Helen (R) University High School
Murphy, Harlan (W) Urbana High School
Terry, Henrietta (G) University High School
Waukegan
Ayre, H. G. (R) Township High School

Indiana (4)
Bloomington
Williams, Kenneth (R) Indiana University
Muncie
Whitcraft, L. H. (R) Ball State Teachers College
Oldenburg
Zieroff, Sister Gertrude Maria (G) St. Louis University
Terre Haute
Kennedy, Kathryn (B) State Teachers College Laboratory School

Iowa (9)
Cedar Falls
Kearney, Dora (B) Teachers College
Clinton
Youngren, Gail (W) Clinton High School
Des Moines
Feglty, Edna (G) Roosevelt High School
Garner
Rommel, Margaret (W) Garner High School
Iowa City
Lane, Ruth (R) University High School
Malvern
Walker, Katherine (W) Malvern High School
Riceville
Short, Lois (W) Riceville High School
Shenandoah
McMullen, Robert (W) High School
Waterloo
McKinley, Eula (W) West High School

Kansas (10)
Hays
Colyer, E. E. (G) Ft. Hays Kansas State College
Kansas City
Cook, Nelle (W) Wyandotte High School
Manhattan
Holroys, Ina (W) Kansas State College
Merriam
Myers, Dessie (W) Shawnee Mission High School
Pittsburg
Curfman, L. W. (W) Kansas State Teachers College
Hill, W. H. (W) Kansas State Teachers College
Shirk, J. A. (G) Kansas State Teachers College
Wichita
Beito, E. A. (R) University of Wichita
Dawson, Lester (W) University of Wichita
Hoare, Arthur (R) University of Wichita

Kentucky (3)
Bowling Green
Gilbert, Dawn (W) Senior High School
Lexington
Latimer, Claiborne (W) University of Kentucky
Shelbyville
Weakley, Rowena (W) Science High School

Louisiana (1)
New Orleans
Mother M. Claire Rivet (W) Ursuline College

Massachusetts (2)
Deerfield
Nichols, Edgar (W) Deerfield Academy
Springfield
Smith, Rolland (R) Classical High School

Michigan (9)
Ann Arbor
Bradshaw, John (W) University of Michigan
Kahol, Nellie (R) Jones School
Karpinski, Louis (R) University of Michigan
Detroit
Smith, Byron (R) Post Intermediate
Flint
Loss, Nellie (R) Flint Secondary Schools
Grand Rapids
Reese, Dora (R) Central High School
Kalamazoo
Everett, John (B) Western State Teachers College
Negaunee
McCling, V. C. (R) Negaunee High School
Sault Ste. Marie
Dow, Ethel (W) Senior High School

Minnesota (6)
Minneapolis
Farnam, Laura (R) Bryant Junior High
Hart, William (R) University of Minnesota
Jackson, Dunham (R) University of Minnesota
Woolsey, Edith (R) Sanford Junior High
Moorhead
Heston, Verna (W) State Teachers College
Northfield
Carlson, C. S. (G) St. Olaf College

Missouri (48)
Cape Girardeau
Knepper, Myrtle (W) State Teachers College
Carthage
Murto, Arria (W) High School
Clayton
Haertter, L. D. (R) John Burroughs School
Langtry, W. A. (W) Clayton High School
Montgomery, Gaylord (R) John Burroughs School
Rule, John (W) Taylor School
Voorhees, Mary (G) Mary Institute
Clinton
Baum, Kathryn (W) Senior High School
Columbia
Butler, Charles (R) University High School
Farmington
Tetlery, Roberta (G) Farmington High School
Jennings
Robertson, C. E. (G) Jennings High School
Kansas City
Briggs, Dorothy (W) Northeast High School
Kirksville
Pemberton, W. S. (R) State Teachers College
Kirkwood
Harris, Elizabeth (G), Guest
Sister Teresa Baiss (G) Ursuline Academy
Lawson
Crossett, Wilma (W) Lawson School
Lexington
Todhunter, Emory (W) Junior Senior High School
St. Charles
Jones, J. V. (G) St. Charles High School
St. Louis
Bailey, Floyd (R) Blewett High School
Barr, Hugh (G) Harris Teachers College
Bishop, Leola (R) Hancheck Senior High School
Brockrieten, Verna (G) Student Harris Teachers College
Davis, Alfred (R) Soldan High
Dietrich, J. J. (W) Riverview Gardens
Dietz, Paul (R) The Principia
Dolans, Ruth (G) Blewart High School
Goorman, Clara (G) Student Harris Teachers College
Grossman, August (R) Roosevelt High School
Koerner, Charles (G) Normandy High School
Link, Mrs. Dean (G) Guest
Osborn, Jesse (G) Harris Teachers College
Reilly, Alice (G) Harris Teachers College
Rosskoff, Myron (W) John Burroughs
Schlierholz, T. (W) Roosevelt High School
Scott, R. E. (W) Central High School
Short, R. L. (W) Ashland
Sister M. Clarita (G) Rosati Kain High School
Sister M. Odilia (G) Rosati Kain High School
Tetley, Ann (G) Ritenour High School
Wallace, Ida (W) Vashon High School
Zacher, Ruth (G) Soldan High School
Zeis, H. C. (R) McKinley High School
St. Louis County
Christian, Wm. (W) Normandy High School
University City
Bruns, Leliaetta (W) University City High School
Howard, Bess (W) University City High School
Turner, Lulu (W) University City High School
Wellston
Horn, Virginia (G) Wellston High School
Webster Groves
Swank, Beulah (W) Webster Groves Junior High School
Nebraska (3)
Lincoln
Congdon, Allan (R) University of Nebraska
Wayne
Boyce, Jessie (G) State Teachers College
Hove, Marie (W) State Teachers College
New Jersey (1)
Montclair
Mallory, Virgil (R) State Teachers College
New York (7)
Alfred
Seidlin, Joseph (R) Alfred University
New York City
Bakst, Aaron (W) Teachers College Columbia
Jablonower, Jas. (B) Fieldston School
Molina, C. C. (G) Bell Telephone Laboratories
Reeve, W. D. (B) Teachers College Columbia
Schlauch, Wm. (B) New York University
Rochester
Betz, William (B) Public Schools
Ohio (12)
Athens
Benz, H. E. (B) Ohio University
Emmert, Robert (W) Ohio University
Morton, R. L. (B) Ohio University
Pickett, Hale (R) Ohio University
Cleveland
MacLearie, Jas. (R) Audubon Junior High School
Miller, A. Brown (R) Fairmount Junior High Training School
Columbus
Fawcett, Harold (R) Ohio State Univ. High School
Oberlin
Cairns, W. D. (R) Oberlin College
Oxford
Christofferson, H. C. (R) Miami University
Shaker Heights
Miller, Florence Brooks (B) Shaker Heights Junior High School
Toledo
Sr. John Baptist (G) Mary Manse College
Sr. V. de Paul (G) Mary Manse College
Oklahoma (5)
Norman
Hassler, J. O. (B) University of Oklahoma
Tulsa
Begley, Kathleen (W) Cleveland High School
Cannon, Kate (W) Roosevelt

Gartman, Stella (W) Horace Mann High School
Reynolds, Ralph (W) Central High School
Pennsylvania (2)
Philadelphia
Stokes, Newton (B) Temple University
Pittsburgh
Olds, E. G. (R) Carnegie Institute of Technology
Tennessee (1)
Nashville
Wren, Lynwood (R) Peabody College
Texas (2)
San Antonio
Darby, Ruth (W) Poe Junior High School
Sister Mary de Tellis (W) Incarnate Word College
Wisconsin (3)
Madison
Hartung, M. L. (R) University of Wisconsin
Milwaukee
Sister Mary Felice (W) Mount Mary College
Racine
Potter, Mary (B) Supervisor of Mathematics

ATTENDANCE BY STATES

	G.	W.	R.	B.	Total
1. Alabama	1	0	0	0	1
2. California	0	0	1	0	1
3. Colorado	1	1	2	0	4
4. District of Columbia	0	0	0	1	1
5. Indiana	1	0	2	1	4
6. Iowa	1	6	1	1	9
7. Illinois	7	24	12	6	49
8. Kansas	2	6	2	0	10
9. Kentucky	0	3	0	0	3
10. Louisiana	0	1	0	0	1
11. Massachusetts	0	1	1	0	2
12. Michigan	0	2	6	1	9
13. Minnesota	1	1	4	0	6
14. Missouri	19	19	10	0	48
15. Nebraska	1	1	1	0	3
16. New Jersey	0	0	1	0	1
17. New York	1	1	1	4	7
18. Ohio	2	1	6	3	12
19. Oklahoma	0	4	0	1	5
20. Pennsylvania	0	0	1	1	2
21. Tennessee	0	0	1	0	1
22. Texas	0	2	0	0	2
23. Wisconsin	0	1	1	1	3
	37	74	53	20	184

CODE: G.—Visitor or guest
W.—Member attending his first annual meeting
R.—Member attending 2–7 annual meetings
B.—Member attending eight or more annual meetings

THE ART OF TEACHING

A NEW DEPARTMENT

Motivating Interest in Problem Solution

By JAMES B. SPRAGUE
Bernardsville High School, Bernardsville, New Jersey

MANY teachers of elementary algebra find that pupils who take a real interest in learning to perform the four fundamental operations with literal expressions, and who are intrigued by the novelty of using negative numbers are strangely unresponsive when introduced to the solution of verbal problems. Even after an appreciation lesson designed to demonstrate that problem solution is a basic aim of the subject, more than one teacher has been greeted by subdued groans, when announcing that the new assignment is the application of some principle to the solution of problems.

The reason for this attitude becomes quite evident when one rationalizes the problem material contained in the average textbook. What possible interest has a child in the solution of a problem concerning the sum of three numbers? Moreover, the type of problem intended to be

practical is often so stated that even the enthusiastic student wonders what real life situation could ever necessitate the solution of one similar. Breslich offers two valid criticisms of problem material: that too many of the customary book problems are of the puzzle type, and that the problems are unreal to the pupils.[1]

Let us consider a typical verbal problem. "A rectangular plot of ground requires 440 yards of fencing. It is three times as long as it is wide. Find the dimensions." Obviously the length and width must have been known in order to determine the given perimeter. Why should not the pupil question the usefulness of such a problem? Breslich suggests restating it to read, "a man wishes to fence in a plot of ground which is to be three times as long as it is wide. He has 440 yards of fencing. What should be the length and width of the plot." Certainly this is a step in the right direction and some of the newer texts have adopted the idea. Something more than mere feasibility however is necessary for any real motivation.

Every individual is fundamentally selfish to the extent that he is essentially interested in matters which concern him personally. All good public speakers recognize this psychological fact and utilize it in addressing an audience. Why not, then, appeal to the individual pupil as a means of motivating interest in problem solution? One way of doing this is by restating verbal problems so that the names of pupils are used. For example the foregoing problem might be stated, "Robert wishes to make a garden and fence it. He has on hand 440 yards of fencing, and plans to make the garden three times as long as it is wide. How long and how wide should it be made?" Different names should be used each day, and the pupils will look eagerly to see if their names appear.

For variation, problems may be chosen that appeal to the interest of particular groups. In this connection it is advisable to use some kind of interest locator, preferably a brief questionnaire, to determine major interests. Problems concerning aviation, games won and lost by athletic teams, et cetera are likely to interest groups of boys; and similar material can be presented for the benefit of the opposite sex. From time to time problems should be proposed which require research by the pupil to secure necessary information.

For motivation not only of the work in verbal problems but also of all work in mathematics, the writer urges every teacher of the subject to read before all classes selected excerpts from "Mathematics in the Training for Citizenship," by David Eugene Smith. This may be found in the Teachers' College Record, May, 1917, Vol. XVIII, No. 3; or on page 390, The Teaching of Mathematics, by J. W. A. Young; Longmans, Green and Company, New York, 1924.

[1] Ernest R. Breslich, Problems in Teaching Secondary School Mathematics, pp. 183, 184, 185. University of Chicago Press, 1931.

The Portland Meeting!

WATCH the May issue of *The Mathematics Teacher* for the program of the summer meeting of the National Council of Teachers of Mathematics at Portland, Oregon from June 27 to June 29 inclusive.

EDITORIALS

New Members of the Council

IT IS gratifying to find Professor Slaught expressing himself (page 184) so strongly about the importance of every mathematics teacher being a member of the National Council of Teachers of Mathematics. There is no doubt that the cause of mathematics in the schools is suffering because of the apathy of some of our teachers. Many of them do not keep informed, do not go to summer school as often as they should, or in other ways improve their outlook. For such teachers and even for some of the rest of us *The Mathematics Teacher* and the Yearbooks constitute a real source of help. How can we reach such teachers? We are constantly receiving letters that indicate that many teachers of mathematics do not even know that a magazine like the *Teacher* exists much less the Yearbooks. Is there any better way to reach these people than for individual teachers who are already members to take a more personal interest in helping us to secure members?

Mrs. Florence Brooks Miller is doing a wonderful piece of work by getting a representative at work in each state. The names of these people will be published in the May issue of the *Teacher*. If every member will get one more member this spring we may soon reach the goal of 10,000 members which Professor Slaught has set.

Obsolete Material in Mathematics

IT IS becoming increasingly clear that algebra in many of our schools is disappearing partly because those who profess to be its friends—teachers of mathematics for the most part—are not awake to the needs of the modern situation in which we now find ourselves. A look at almost any one of the newer text books on algebra will reveal page after page of traditional material that can not be justified by desirable objectives. Unless classroom teachers of algebra lead in insisting that such material be omitted from their courses, if not from the text books, worse days are ahead. Similar statements might be made with respect to other content material in other courses in mathematics, but it is this "deadwood" in algebra and excessive formalism in teaching that is responsible for the most of our trouble.

We cannot sit by as some people would have us believe and hide the fact that mathematics is under fire by keeping still or even by setting up programs for better teaching that will convince the uninformed of our worthy purpose. Before we can show what a wonderful subject mathematics is, how it can be successfully taught, and before we can make it secure in the schools, we must eliminate much of the traditional material, reorganize what is left so that it can be made more useful to larger groups of children, and introduce more material of the right sort that can be justified by the requirements of 1936 instead of the eighteenth century.

ON TO PORTLAND!

IN OTHER PERIODICALS

By NATHAN LAZAR
Alexander Hamilton High School, Brooklyn, New York

Algebra

1. Blazej, Richard. *The graph of a linear equation.* High Points. October 1935, pp. 70–2.

A detailed description of a method of teaching graphs to a class in intermediate algebra. The delta notation is used.

2. Stellard, Burton J. and Douglass, Harl R. *An experimental study of two plans of supervised study in first year algebra.* School Science and Mathematics. 36: 78–81. January 1936.

The authors report on an investigation "planned and carried on in an effort to throw some light on the question of whether the long unit assignment plan of supervised study is superior to the divided period daily assignment plan." The conclusion is that, "the type of large unit plan of supervised study employed by Mr. Stellard was found to be superior to his plan of daily recitation plan sections composed of superior pupils, though no significant difference was found between the two plans for sections of pupils of average ability." A bibliography or five items is included.

Arithmetic

1. Brueckner, Leo J. *Persistency of error as a factor in diagnosis.* Education. 56: 140–4. November 1935.

"Varability of response is an important factor in educational diagnosis that is not often taken into account in analyzing the results of arithmetic diagnostic tests." The usual way of determining the reliability of tests often fails to take into account the fact "that while the two scores of an individual in the test may be practically the same, the specific items incorrect on the two tests and the kinds of errors made may be quite different. An analysis of the specific errors on one performance of a test may therefore lead to a wholly invalid diagnosis." A report is given of the results of a series of experiments bearing on this point.

2. Grossnickle, F. E. *Practice material in the estimation of the quotient in long division found in current arithmetic workbooks.* Journal of Educational Research. 28: 668–88. May 1935.

The study was made "to determine the amount of variation found in the division examples given in the arithmetic workbook series which has a separate booklet for each grade from the third to the eighth inclusive." The author concludes that "to a large extent, the amount and the type of practice in long division which a pupil will receive from one of these workbooks depends upon the series which he uses. . . . There is a pronounced need for an experimental study which will show the approximate number of estimations essential for pupil mastery of a given division difficulty."

3. Watson, Stanley A. *Arithmetic, Junior third to Senior fourth, Grades V–VIII.* School (Toronto, Elementary Edition). 24: 299–303. December 1935.

A continuation of a series of articles on the same subject. In this article the author deals particularly with causes of error in problem solving, and with problems in percentage.

Miscellaneous

1. Adams, L. J. *Better Junior College Mathematics.* The Junior College Journal. 6: 140–42. December 1935.

A plea to the teacher of mathematics in the junior colleges to examine the predicament of mathematics, to readjust their attitudes and philosophy, and to coöperate with the other departments of the college.

2. Breslich, E. R. *Integration of secondary-school mathematics and science.* School Science and Mathematics. 36: 58–67. January 1936.

After an historical introduction to the problem, Professor Breslich proceeds to an analysis of the difficulties inherent in the attempts at its solution. He outlines nine steps to be taken by schools planning to experiment with an integrated program in science and mathematics, and points out the advantages to be derived from the adoption of the plan advocated.

3. Greenfield, Samuel C. *Failure in Mathematics: A problem in mental hygiene.* High Points. December 1935, pp. 16–22.

A very sympathetic and interesting account of the psychological causes underlying failure in mathematics. Some of the cases reported in this article are taken from a fuller account by the same author, in collaboration with Dr. Samuel C. Karlan, that appeared in Mental Hygiene, 18: 611–20, October 1934.

4. Kroll, Abraham. *The coöperative tests in mathematics at the De Witt Clinton High School.* High Points. October 1935, pp. 29–31.

A report on a study made to determine the advisability of giving the tests of the Coöperative Test Service in place of the regular school examinations. The writer concludes that so far as De Witt Clinton High School is concerned "midterm examination marks or teachers' final marks or a combination of both is more effective for determining what a pupil will do on the regents examination in Intermediate Algebra, in Plane Geometry or in Plane Trigonometry, than the 1933 form of the Coöperative Tests in these subjects."

5. Lange, Luise. *On fingerprints in number words.* School Science and Mathematics. 36: 13–19. January 1936.

The author points out the very important fact that "our number system is decimal not only because we write numbers decimally, but because we talk decimally as well. Not only our number symbols are patterned after our ten fingers, but our number words as well."

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NEWS NOTES

At Teachers College, Columbia University, from July 7th to August 14th, in addition to the major course on the teaching and supervision of mathematics and one on the teaching of algebra given by Professor Reeve, the following courses will be offered. By Miss Sutherland: Teaching arithmetic in primary grades, teaching arithmetic in intermediate grades, and professionalized subject matter in junior high school mathematics. By Professor Shuster: Modern business arithmetic and field work in mathematics. By Dr. Sanford: Teaching algebra in junior high schools, and the history of mathematics. By Mr. Smith: Teaching geometry in secondary schools and a demonstration class in plane geometry. By Dr. Swenson: Professionalized subject matter in senior high school mathematics, and a demonstration class in eleventh year mathematics. By Dr. Wolff, visiting professor from Germany: Teaching mathematics in the secondary schools of Germany and the correlation of secondary school mathematics with science and art.

Dr. E. R. Hedrick, the chairman of the Department of Mathematics in the University of California at Los Angeles, is scheduled for two courses in the teaching of mathematics in the School of Education of New York University Summer School, July 6th–August 14th. One course is the teaching of senior high school mathematics and the other the teaching of College Mathematics. Professor Drushel will give a course on the content of junior high school mathematics and one on the teaching of problem solving in elementary mathematics. Mrs. Pieper will give a course on the teaching and supervision of arithmetic in the lower grades.

Mr. A. E. Robinson of Newtown High School at Elmhurst, L. I. pays a great tribute to the Tenth Yearbook of the National Council of Teachers of Mathematics on "The Teaching of Arithmetic" when he writes "There ought to be a law requiring every teacher and supervisor of arithmetic in the whole country to read thoroughly *twice* the Tenth Yearbook of the National Council. It certainly drives home some hard facts."

On November 9, 1935, the Association of Mathematics Teachers of New Jersey held their annual meeting at Atlantic City. The theme of the meeting was "The New Mathematics Requirement of the College Entrance Board." The point of view of the commission which framed the requirements was given by Professor Arnold Dresden of Swarthmore College. The point of view of the College was given by Professor Robert M. Walter of the New Jersey College for Women. The point of view of the secondary school was given by Mr. Rolland R. Smith of the Classical High School of Springfield, Massachusetts. The discussion was led by Professor J. R. Kline of the University of Pennsylvania.

The Chairman of the Mathematics departments of the New York City senior high schools held their annual open luncheon at the Hotel Astor on Saturday February 15, 1936. About 400 were present. Mr. Shanholt of Abraham Lincoln High School presided and Julius Zimmerman was in charge of arrangements. Dr. Arnold Dresden of Swarthmore College was the guest speaker. His topic was "The Relation of Mathematics to the Needs of the Modern World."

The First Summer Meeting of the National Council of Teachers of Mathematics was held at Denver, Colorado on June 29–July 1, 1935

The convention opened with an informal luncheon in honor of Dr. E. R. Hedrick of the University of California at Los Angeles at the Argonaut Hotel at 12 o'clock Saturday, June 29.

First Program Meeting. Saturday, June 29, 2:00 P.M.

Vice-president A. R. Congdon presiding.

1. Welcome to Colorado. H. W. Charlesworth, Chairman of the Local Committee Denver, Colo.
2. The Trend toward Fusion Mathematics as shown by an Analysis of Junior High School texts. Carl S. Thomas, Chadron, Nebr.
3. A High School Teacher's View on the Training of Teachers of Secondary Mathematics. Martha Hildebrandt, Maywood, Ill.
4. Geometric Design (Illustrated). A. Bruce Ewer, Johnstown, Colo.
5. Announcements about Sunday's Outing by Mr. H. W. Charlesworth, Mrs. Ruby

Shute Flannery, and Miss Margaret H. Aylard.

A telegram from President J. O. Hassler giving greetings and best wishes for the success of the convention was read by Dr. Congdon.

This program was followed by a discussion. 102 attended the meeting.

Saturday Banquet. Argonaut Hotel, June 29, 6:30 P.M.

This banquet was attended by 65 members and guests, seated at one large table. Dinner music was furnished by the Art Trio of Denver. At the conclusion of the banquet Vice-President A. R. Congdon introduced to the audience the three members of the Board of Directors who were present: Miss Edith Woolsey, acting secretary, Miss Martha Hildebrandt, and Miss Mary A. Potter. Violin selections by Mr. George Wagner, East High School, Denver, were rendered. Dr. Congdon Introduced Professor Charles A. Hutchinson of the University of Colorado who presented the speaker of the Evening, Dr. Earle R. Hedrick of the University of California at Los Angeles. Dr. Hedrick gave a very timely, practical, and inspiring address on the subject, "Crisis in Education, Economics, and Mathematics."

Echo Lake Outing—Mountain Trip and Picnic, Sunday, June 30, 9:45 A.M.

The all day outing at Echo lake was attended by 128. A caravan made up of more than forty automobiles, with police escort, left the Argonaut Hotel at 10:00 A.M. and arrived at Echo Lake about 1:00. Transportation and a barbecue dinner with food in endless variety were furnished by the Local Committee. The gorgeous mountain scenery, the Thrilling roads with Switch-backs and hair-pin curves, the jewel-like lake, and the unequalled hospitality of our Colorado hosts and hostesses made this day one to be remembered always.

Breakfast Meeting. Argonaut Hotel, Monday, July 1, 8:00 A.M.

A breakfast at the Argonaut Hotel, attended by 51, preceded the morning meeting. After the breakfast the group adjourned to the assembly room for the morning's program. The total attendance at this meeting was 79.

Miss Edith Woolsey presided.

1. The Human Side of Mathematics. Miss Mary A. Potter, supervisor of mathematics at Racine, Wisconsin.
2. Dr. Congdon explained the work of the Council and made a plea for more members and a wider use of the yearbooks.
3. Mrs. Ruby Shute Flannery introduced a resolution recommending that the officers of the Council consider plans to affiliate with the N.E.A. She explained the difficulties the Denver Group had in getting any advertising or cooperation from the N.E.A. for this summer meeting. After some discussion the vote to support this resolution was unanimous.
4. Mr. Charlesworth urged the Council to make great efforts to have local mathematics clubs affiliate with the National Council. He gave as his reason the great impetus which the Denver club got from the affiliation. He also urged the Council to cooperate with the local clubs to make their work more effective.
5. Miss Alfhild Alenius reminded those present of the mathematics exhibits at the East High School and at the Morey Junior High.

Joint Conference—Department of Secondary Education in Cooperation with the National Council. East High School, Monday July 1, 3:00 P.M.

Central Topic: How can mathematics contribute to the solution of present Youth problems?

Vice-President A. R. Congdon presiding.

1. Paul Beck of Tulsa, Okla., read a paper by L. W. Lavengood of Tulsa, Okla., who was unable to be present.
2. G. W. Finley, State College of Education, Greeley, Colo.
3. Spirited discussion led by J. C. Stearns, University of Denver, Denver, Colo.

This meeting was attended by 215.

The first summer meeting adjourned.

SUMMARY OF ATTENDANCE

State	G.	W.	R.	B.	Total
California	1	1	1	0	3
Colorado	45	37	5	0	87
Illinois	2	0	0	1	3
Iowa	1	0	0	0	1
Kansas	2	2	0	0	4
Maryland	2	0	0	0	2
Michigan	0	0	1	0	1
Minnesota	1	0	1	0	2
Missouri	1	1	0	0	2
Nebraska	5	4	1	0	10
North Dakota	1	0	0	0	1
New York	2	0	0	0	2
Oregon	1	1	0	0	2
Texas	1	0	0	0	1
Washington	1	0	0	0	1
Wisconsin	0	1	0	1	2
Totals	66	47	9	2	124

CODE:

G.—Visitor or guest
W.—Member attending his first meeting.
R.—Member attending 2–7 meetings.
B.—Member attending eight or more meetings.

EDITH WOOLSEY

Ginn and Company have recently announced that Dr. Burdette R. Buckingham has been admitted to partnership in the firm. Since 1928, when he resigned as Director of the Bureau of Education Research at Ohio State University to join the Editorial Department of Ginn and Company, he has been in charge of the elementary and junior high school publications of that firm.

Doctor Buckingham (B.A. and M.A., Wesleyan University, Connecticut, Pd.B., State Normal College, Albany, New York, Ph.D., Teachers College, Columbia; Ed.D., Miami University) has had long and varied experience in the field of education. After several years as teacher of mathematics and principal in schools in New York he became successively chief statistician in the office of the city superintendent of schools, New York; educational statistician for the State Board of Education at Madison, Wisconsin; professor of education and director of the Bureau of Educational Research at the University of Illinois; and director of the Bureau of Educational Research at Ohio State University. He has also been a lecturer on education and a member of the faculty at the Harvard Graduate School of Education.

Doctor Buckingham has been a director of the National Society for the Study of Education, a member of the American Statistical Association and of the National Research Council, and a fellow of the Royal Statistical Society, London, He is now a life member of the National Education Association, a member of the Educational Research Association, of which he has twice been president, and a member of the American Psychological Association, of the National Council of Teachers of English, of the National Council of Teachers of Geography, and of the National Council of Teachers of Mathematics.

Among his publications are the following: Spelling Ability—Its Measurement and Distribution, Supply and Demand in Teacher Training, Research for Teachers, and (with W. J. Osburn) the Buckingham-Osburn Searchlight Arithmetics. Doctor Buckingham has also written numerous articles on various phases of educational research and has been editor of the *Journal of Educational Research* (1920–1928) and of the *Educational Research Bulletin* (1922–1928).

The following review of the Tenth Yearbook of the National Council of Teachers of Mathematics on the "Teaching of Arithmetic" appeared in *Educational Method* for February 1936: "The tenth yearbook of the National Council of Teachers of Mathematics contains thirteen chapters on various aspects of the teaching of arithmetic. Each writer worked alone but there is some cross reference, and the chapter by David Eugene Smith is largely by way of commentary on the nine chapters that precede it.

"As a whole this yearbook comes to grips with its subject in much more satisfying fashion than did that of the National Society for the Study of Education, published in 1930. It faces the issues—not all of them, to be sure, but several of them. Brownell, for example, attacks both the drill and the incidental methods. Buckingham outlines the information that should be in the course and for the most part isn't. Hanna, with the help of others, presents the opportunities for the use of arithmetic in an activity program. Dr. Smith, in his usual trenchant style, portrays the future, in which children will be asked to learn only the arithmetic that is actually useful, with the terminology of everyday life in place of the archaic jargon now imposed upon them.

"Not all of the contributions are as vital and forward-looking as these, but taken as a whole the tone of the book is distinctly modern."

The Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics and the Louisiana-Mississippi Section of the Mathematical Association of America met on March 13 and 14 at the State Teachers College in Hattiesburg, Miss. Professor J. O. Hassler was sent by the Association and spoke on the topics "An Evaluation and Comparison of Objective and Subjective Tests in Mathematics" and "The Spirit of Discovery—A Motivating Power in Learning Mathematics." Professor Hassler is the retiring president of the National Council of Teachers of Mathematics.

The third dinner meeting of the Men's Mathematics Club was held at the Central Y.M.C.A. on Friday Dec. 13, 1935. C. M. Austin reported on the State High School Conference at Urbana, M. D. Oestreicher on the Central Association of Science and Mathematics Annual meeting at Chicago, and Alexander Ritter gave a description of "The Whitehead Fish Torpedo and some Thermodynamical Theories."

The fourth dinner meeting of the Men's Mathematics Club of Chicago was held at the Central Y.M.C.A. on January 17. Mr. Everett W. Owen and Mr. Butler Laughlin were the guest speakers.

The annual joint dinner meeting of the Men's and Women's Mathematics Clubs of Chicago was held at the Chicago Woman's Club on Friday evening February 7, 1936.

The menu and program follow:

Menu

Square Meals
at Round Tables

Cube Bouillon with
Rectangular Crackers
or
Blue Points
on Half Shell

Linear Celery
Cylindrical Pickles
Elliptical Olives
Conical Radishes

Squaretail Trout
or
Round Steak
from an Angular Cow

Prismatic Potatoes
Spherical Peas

"Let x" Salad
with 1000 Dressing

Plane Rolls
with Solid Butter

Segment of π with
Triangular Cheese

Circle Coffee

Acute Indigestion

$1.10 Q.E.D.

Programme

Theorem:

Harts and Flowers
are a Dandeline

Given:

Professor
Walter W. Hart
of the
University of Wisconsin

To Prove:

"Forward Mathematics"
with Tulips

Proof

1: ♡ *will give us*
Poin(t)settias

2: ♡ *will give us*
Forget-me-not(e)s

3: Therefore ♡ *with*
Tulips will give us a
Dandeline.

Q.E.D.

Miss Eleanor Booker of Oak Park High School, president of the Women's Club presided. There were 150 people present. Mr. Wm. C. Krathwohl is president of the Men's Club.

Professor W. D. Reeve of Teachers College Columbia University was the guest speaker at a special called joint dinner meeting of the Men's and Women's Mathematic's Clubs of Chicago at the Central Y.M.C.A. on Friday evening Feb. 21. His topic was "Pressing Curriculum Problems in Teaching Mathematics."

Section 19 (Mathematics) of the New York Society for the Experimental Study of Education held its fourth dinner meeting on Saturday Feb. 8. Professor A. A. Bennett of Brown University was the guest speaker. His topic was "A Course in Statistics for High Schools."

The Mathematical Association held its annual meeting at The Institute of Education in London on Jan. 2nd and 3rd.

The following program was given:

Thursday P.M.

Address—*Progress*
President A. W. Siddons
Address—*The Physics of Sport*
Professor Sir Gilbert Walker, F. R. S.

Friday A.M.

Rider Work in Geometry
Discussion opened by Mr. E. H. Lockwood followed by Mr. F. S. Tongue.

Work for University Entrance Scholarships.
Discussion by Messrs. A. Robson, G. L. Parsons, M. H. A. Newman, and A. H. Wilson.

Friday P.M.

The Rehabilitation of Differentials.
Professor G. F. J. Temple

Exhibition of Mathematical Films.
Miss M. Punnett.

The fifth dinner meeting of Section 19 (Mathematics) of the New York Society for the Experimental Study of Education will be held at the Men's Faculty Club of Columbia University on Saturday evening April 4 at 6:30 o'clock sharp. There will be a panel discussion on the topic "The New Mathematics Requirements of the College Entrance Examination Board and their Probable Effect Upon the Teaching of Mathematics in the Schools." Professor C. B. Upton, of Teachers College, will be the panel leader. At the last dinner meeting of Section 19 on May 2, Dr. E. R. Breslich of the University of Chicago will be the guest speaker. His topic will be "The Mental Processes of Algebra." The price of the dinner in either case above is $1.25. Send reservations to W. D. Reeve, 525 West 120th St., New York, N. Y.

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